6. Since the product $\begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{5} & -\frac{1}{10} \end{bmatrix} = \begin{bmatrix} 0+1 & \frac{1}{2}-\frac{1}{2} \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$ and the product

$$\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{5} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0 \\ \frac{1}{5}-\frac{2}{10} & 1-0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2,$$

the matrices are inverses of each other.

11. First, we augment the matrix with I_2; then we use row operations to obtain the reduced row echelon form of the matrix.

$$\left[\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow[R_1=r_1-r_2]{} \left[\begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow[R_2=-2r_1+r_2]{} \left[\begin{array}{cc|cc} 1 & 2 & 1 & -1 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

$$\xrightarrow[R_1=-2r_2+r_1]{} \left[\begin{array}{cc|cc} 1 & 0 & 5 & -7 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

Since the identity matrix I_2 is on the left side, the matrix on the right is the inverse.

$$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

12. First, we augment the matrix with I_2; then we use row operations to obtain the reduced row echelon form of the matrix.

$$\left[\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] \xrightarrow[R_1=r_1-r_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 3 & 1 & 0 & 1 \end{array}\right] \xrightarrow[R_2=-3r_1+r_2]{} \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & 4 \end{array}\right]$$

Since the identity matrix I_2 is on the left side, the matrix on the right is the inverse.

$$\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

19. First, we augment the matrix with I_3, and then we use row operations to obtain the reduced row echelon form of the matrix.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{R_2=-3r_1+r_2 \\ R_3=-2r_1+r_3}]{} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -4 & 3 & -3 & 1 & 0 \\ 0 & -5 & 6 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2=-\frac{1}{4}r_2]{} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & -5 & 6 & -2 & 0 & 1 \end{array}\right] \xrightarrow[\substack{R_1=-r_2+r_1 \\ R_3=5r_2+r_3}]{} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{9}{4} & \frac{7}{4} & -\frac{5}{4} & 1 \end{array}\right]$$

$$\xrightarrow[R_3=\frac{4}{9}r_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{4} & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{7}{9} & -\frac{5}{9} & \frac{4}{9} \end{array}\right] \xrightarrow[\substack{R_1=\frac{1}{4}r_3+r_1 \\ R_2=\frac{3}{4}r_3+r_2}]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{4}{9} & \frac{1}{9} & \frac{1}{9} \\ 0 & 1 & 0 & \frac{4}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{7}{9} & -\frac{5}{9} & \frac{4}{9} \end{array}\right]$$

Since the identity matrix I_3 is on the left side, the matrix on the right is the inverse.

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{4}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{4}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{9} & -\frac{5}{9} & \frac{4}{9} \end{bmatrix}$$

20. First, we augment the matrix with I_3; then we use row operations to obtain the reduced row echelon form of the matrix.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{R_2=-2r_1+r_2 \\ R_3=-r_1+r_3}]{} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2=-r_2]{} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_1=-r_2+r_1]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2=-r_3+r_2]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Since the identity matrix I_3 is on the left side, the matrix on the right is the inverse.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

26. First, we augment the matrix with I_2; then we use row operations to obtain the reduced row echelon form of the matrix.

$$\left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 3 & -6 & 0 & 1 \end{array}\right] \xrightarrow{R_1=-r_1} \left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 3 & -6 & 0 & 1 \end{array}\right] \xrightarrow{R_2=-3r_1+r_2} \left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

The 0s in row 2 indicate that we cannot get the identity matrix. This tells us that the original matrix has no inverse.

29. Since the matrix has a row of zeros, it has no inverse.

36. To find the inverse we augment the matrix with I_2, and then use row operations to obtain the reduced row echelon form.

$$\left[\begin{array}{cc|cc} 4 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_1=\frac{1}{4}r_1} \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{4} & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2=-2r_1+r_2} \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{array}\right]$$

The 0s in the row 2 tell us we cannot get the identity matrix. This indicates that the original matrix has no inverse.

44. To solve the system $\begin{cases} 3x+7y=-4 \\ 2x+5y=-3 \end{cases}$ we define $A=\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$, and $B=\begin{bmatrix} -4 \\ -3 \end{bmatrix}$.

The solution to the system is $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

So, $x=1$ and $y=-1$ or $(1,-1)$.

50. To solve $\begin{cases} x+\ y-\ z=\ 6 \\ 3x-\ y \qquad = \ 8 \\ 2x-3y+4z=-3 \end{cases}$ we define $A=\begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B=\begin{bmatrix} 6 \\ 8 \\ -3 \end{bmatrix}$.

The solution to the system is $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 8 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{4}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{4}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{7}{9} & -\frac{5}{9} & \frac{4}{9} \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{29}{9} \\ \frac{5}{3} \\ -\frac{10}{9} \end{bmatrix}$$

So, $x=\frac{29}{9}$, $y=\frac{5}{3}$, and $z=-\frac{10}{9}$ or $\left(\frac{29}{9},\frac{5}{3},-\frac{10}{9}\right)$.