8. The problem is in standard form since both conditions are met.

9. The problem is not in standard form since variables x_2 and x_3 are not given as nonnegative.

14. The problem is not in standard form. The constraint $2x_1 - x_2 \geq 1$ is not a linear expression that is less than or equal to a positive constant.

18. The problem can be modified so it is in standard form. Multiply the first constraint by –1. The problem becomes

Maximize

$$P = 2x_1 + 3x_2$$

subject to the constraints

$$\begin{aligned} 4x_1 - 2x_2 &\leq 8 \\ x_1 - \ \ x_2 &\leq 6 \\ x_1 \geq 0 \quad x_2 &\geq 0 \end{aligned}$$

19. The maximum problem cannot be modified so to be in standard form.

23. We write the objective function as

$$P - 2x_1 - x_2 - 3x_3 = 0$$

subject to the constraints

$$\begin{aligned} 5x_1 + 2x_2 + \ \ x_3 + s_1 \qquad\qquad &= 20 \\ 6x_1 - \ \ x_2 + 4x_3 \qquad + s_2 \qquad &= 24 \\ x_1 + \ \ x_2 + 4x_3 \qquad\qquad + s_3 &= 16 \end{aligned}$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \quad s_1 \geq 0 \quad s_2 \geq 0 \quad s_3 \geq 0$$

The initial tableau is

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccccc} P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 5 & 2 & 1 & 1 & 0 & 0 & 20 \\ 0 & 6 & -1 & 4 & 0 & 1 & 0 & 24 \\ 0 & 1 & 1 & 4 & 0 & 0 & 1 & 16 \\ \hline 1 & -2 & -1 & -3 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

26. We write the objective function as

$$P - 2x_1 - 3x_2 = 0$$

subject to the constraints

$$\begin{aligned} 1.2x_1 - 2.1x_2 + s_1 \qquad\qquad &= 0.5 \\ 0.3x_1 + 0.4x_2 \qquad + s_2 \qquad &= 1.5 \\ x_1 + \quad x_2 \qquad\qquad + s_3 &= 0.7 \end{aligned}$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad s_1 \geq 0 \quad s_2 \geq 0 \quad s_3 \geq 0$$

The initial tableau is

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{ccccccc} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 1.2 & -2.1 & 1 & 0 & 0 & 0.5 \\ 0 & 0.3 & 0.4 & 0 & 1 & 0 & 1.5 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0.7 \\ \hline 1 & -2 & -3 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

32. We modify the problem by writing each linear constraint as an inequality less than or equal to a positive constant. The modified maximum problem in standard form is

Maximize

$$P = 2x_1 + 4x_2 + x_3$$

subject to the constraints

$$\begin{aligned} 2x_1 + 3x_2 - \ x_3 &\le 8 \\ 3x_1 - \ x_2 + 2x_3 &\le 12 \\ 2x_1 + \ x_2 + \ x_3 &\le 10 \\ x_1 \ge 0,\ x_2 \ge 0,\ x_3 &\ge 0 \end{aligned}$$

We then introduce slack variables and set up the initial simplex tableau.

$$\begin{aligned} P - 2x_1 - 4x_2 - \ x_3 \qquad\qquad &= 0 \\ 2x_1 + 3x_2 - \ x_3 + s_1 \qquad &= 8 \\ 3x_1 - \ x_2 + 2x_3 \quad + s_2 \quad &= 12 \\ 2x_1 + \ x_2 + \ x_3 \qquad + s_3 &= 10 \\ x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ s_1 \ge 0,\ s_2 &\ge 0,\ s_3 \ge 0 \end{aligned}$$

The initial tableau is

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccccc} P & x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 2 & 3 & -1 & 1 & 0 & 0 & 8 \\ 0 & 3 & -1 & 2 & 0 & 1 & 0 & 12 \\ 0 & 2 & 1 & 1 & 0 & 0 & 1 & 10 \\ \hline 1 & -2 & -4 & -1 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

35. Pivoting: Step 1: Divide each entry in the *pivot row* by the *pivot element*.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 2 & 1 & 0 & 300 \\ 0 & \boxed{3} & 2 & 0 & 1 & 480 \\ \hline 1 & -1 & -2 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \begin{array}{c} \text{BV} \\ s_1 \\ \to x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 2 & 1 & 0 & 300 \\ 0 & 1 & \frac{2}{3} & 0 & \frac{1}{3} & 160 \\ \hline 1 & -1 & -2 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Step 2: Obtain 0s elsewhere in the *pivot column* by performing row operations using the revised *pivot row*.

$$\xrightarrow[R_3 = R_2 + r_3]{R_1 = -R_2 + r_1} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 0 & \frac{4}{3} & 1 & -\frac{1}{3} & 140 \\ 0 & 1 & \frac{2}{3} & 0 & \frac{1}{3} & 160 \\ \hline 1 & 0 & -\frac{4}{3} & 0 & \frac{1}{3} & 160 \end{array}\right] \end{array}$$

The system of equations corresponding to the new tableau is

$$\begin{cases} s_1 = -\dfrac{4}{3}x_2 + \dfrac{1}{3}s_2 + 140 \\ x_1 = -\dfrac{2}{3}x_2 - \dfrac{1}{3}s_2 + 160 \\ P = \ \dfrac{4}{3}x_2 - \dfrac{1}{3}s_2 + 160 \end{cases}$$

The current values are $P = 160$, $x_1 = 160$, and $s_1 = 140$.

36. Pivoting: Step 1: Divide each entry in the *pivot row* by the *pivot element*.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 4 & 1 & 0 & 100 \\ 0 & \boxed{2} & 5 & 0 & 1 & 50 \\ \hline 1 & -2 & -1 & 0 & 0 & 0 \end{array}\right] \end{array} \quad \begin{array}{c} \text{BV} \\ s_1 \\ \to x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 4 & 1 & 0 & 100 \\ 0 & 1 & \frac{5}{2} & 0 & \frac{1}{2} & 25 \\ \hline 1 & -2 & -1 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Step 2: Obtain 0s elsewhere in the *pivot column* by performing row operations using the revised *pivot row*.

$$\xrightarrow[\substack{R_1 = -1R_2 + r_1 \\ R_3 = 2R_2 + r_3}]{} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} P & x_1 & x_2 & s_1 & s_2 & \text{RHS} \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 0 & \frac{3}{2} & 1 & -\frac{1}{2} & 75 \\ 0 & 1 & \frac{5}{2} & 0 & \frac{1}{2} & 25 \\ \hline 1 & 0 & 4 & 0 & 1 & 50 \end{array}\right] \end{array}$$

The system of equations corresponding to the new tableau is

$$\begin{cases} s_1 = -\dfrac{3}{2}x_2 + \dfrac{1}{2}s_2 + 75 \\ x_1 = -\dfrac{5}{2}x_2 - \dfrac{1}{2}s_2 + 25 \\ P = -4x_2 - s_2 + 50 \end{cases}$$

The current values are $P = 50$, $x_1 = 25$, and $s_1 = 75$.