11. The minimum problem is not in standard form. Condition 3 is not met; the objective function has a negative coefficient.

14. The minimum problem is not in standard form. Condition 3 is not met; the objective function has a negative coefficient.

15. **STEP 1** The problem is in standard form.

STEP 2 The matrix with the objective function in the bottom row:

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 6 \\ 2 & 3 & 0 \end{array}\right]$$

STEP 3 The matrix with the rows and columns interchanged:

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 1 & 3 & 3 \\ 2 & 6 & 0 \end{array}\right]$$

STEP 4 The corresponding maximum problem in standard form:

Maximize

$$P = 2y_1 + 6y_2$$

subject to the constraints

$$y_1 + 2y_2 \le 2$$
$$y_1 + 3y_2 \le 3$$
$$y_1 \ge 0 \quad y_2 \ge 0$$

This maximum problem is the dual of the minimum problem.

17. **STEP 1** The problem is in standard form.
STEP 2 The matrix with the objective function in the bottom row:

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 1 & 0 & 4 \\ 3 & 1 & 1 & 0 \end{array}\right]$$

STEP 3 The matrix with the rows and columns interchanged:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 5 & 4 & 0 \end{array}\right]$$

STEP 4 The corresponding maximum problem in standard form:

Maximize

$$P = 5y_1 + 4y_2$$

subject to the constraints

$$\begin{aligned} y_1 + 2y_2 &\le 3 \\ y_1 + \ \ y_2 &\le 1 \\ y_1 \qquad &\le 1 \\ y_1 \ge 0 \quad y_2 &\ge 0 \end{aligned}$$

This maximum problem is the dual of the minimum problem.

22. **STEP 1** Write the dual problem.

The matrix: $\begin{array}{c} \begin{matrix} x_1 & x_2 & \end{matrix} \\ \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 0 \end{array}\right] \end{array}$; its transpose: $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 1 & 4 \\ 3 & 4 & 0 \end{array}\right]$;

Maximize

$$P = 3y_1 + 4y_2$$

subject to the constraints

$$\begin{aligned} y_1 + 2y_2 &\le 3 \\ y_1 + \ \ y_2 &\le 4 \\ y_1 \ge 0 \quad y_2 &\ge 0 \end{aligned}$$

STEP 2 Set up the initial tableau and use the simplex method to solve the dual problem.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{matrix} P & y_1 & y_2 & s_1 & s_2 & \text{RHS} \end{matrix} \\ \left[\begin{array}{cccccc|c} 0 & \boxed{1} & 2 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 1 & 4 \\ \hline 1 & -3 & -4 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow[R_3 = 3R_1 + r_3]{R_2 = -R_1 + r_2} \begin{array}{c} \text{BV} \\ y_1 \\ s_2 \\ P \end{array} \begin{array}{c} \begin{matrix} P & y_1 & y_2 & s_1 & s_2 & \text{RHS} \end{matrix} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 2 & 1 & 0 & 3 \\ 0 & 0 & -1 & -1 & 1 & 1 \\ \hline 1 & 0 & 2 & 3 & 0 & 9 \end{array}\right] \end{array}$$

STEP 3 This is the final tableau. From it we read that the minimum cost $C = 9$ is obtained when $x_1 = 3$ and $x_2 = 0$.

23. **STEP 1** Write the dual problem.

The matrix: $\begin{bmatrix} 1 & 1 & | & 4 \\ 3 & 4 & | & 12 \\ 6 & 3 & | & 0 \end{bmatrix}$ (columns x_1, x_2); its transpose: $\begin{bmatrix} 1 & 3 & | & 6 \\ 1 & 4 & | & 3 \\ 4 & 12 & | & 0 \end{bmatrix}$;

Maximize

$$P = 4y_1 + 12y_2$$

subject to the constraints

$$y_1 + 3y_2 \le 6$$
$$y_1 + 4y_2 \le 3$$
$$y_1 \ge 0 \qquad y_2 \ge 0$$

STEP 2 Set up the initial tableau and use the simplex method to solve the dual problem.

BV	P	y_1	y_2	s_1	s_2	RHS
s_1	0	1	3	1	0	6
s_2	0	[1]	4	0	1	3
P	1	–4	–12	0	0	0

$\xrightarrow[R_3 = 4R_2 + r_3]{R_1 = -R_2 + r_1}$

BV	P	y_1	y_2	s_1	s_2	RHS
s_1	0	0	–1	1	–1	3
y_1	0	1	4	0	1	3
P	1	0	4	0	4	12

STEP 3 This is the final tableau. From it we read that the minimum cost $C = 12$ is obtained when $x_1 = 0$ and $x_2 = 4$.