5. Rewrite the constraints:

$$\begin{aligned} x_1 + \ \ x_2 &\le 12 \\ -5x_1 - 2x_2 &\le -36 \\ -7x_1 - 4x_2 &\le -14 \\ x_1 \ge 0,\ x_2 &\ge 0 \end{aligned}$$

Introduce slack variables:

$$\begin{aligned} x_1 + \ \ x_2 + s_1 \qquad\qquad &= 12 \\ -5x_1 - 2x_2 \qquad + s_2 \qquad &= -36 \\ -7x_1 - 4x_2 \qquad\qquad + s_3 &= -14 \\ x_1 \ge 0,\ x_2 \ge 0,\ s_1 \ge 0,\ s_2 \ge 0,\ s_3 &\ge 0 \end{aligned}$$

We now set up the initial tableau and use the alternate pivoting method as long as there are negative entries in the RHS. When all the entries in the RHS are positive, we use the standard way of choosing a pivot.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 12 \\ 0 & \boxed{-5} & -2 & 0 & 1 & 0 & -36 \\ 0 & -7 & -4 & 0 & 0 & 1 & -14 \\ \hline 1 & -3 & -4 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow[\text{Strategy}]{\text{Alternative Pivoting}} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 0 & \boxed{\frac{3}{5}} & 1 & \frac{1}{5} & 0 & \frac{24}{5} \\ 0 & 1 & \frac{2}{5} & 0 & -\frac{1}{5} & 0 & \frac{36}{5} \\ 0 & 0 & -\frac{6}{5} & 0 & -\frac{7}{5} & 1 & \frac{182}{5} \\ \hline 1 & 0 & -\frac{14}{5} & 0 & -\frac{3}{5} & 0 & \frac{108}{5} \end{array}\right] \end{array}$$

$$\xrightarrow[\text{Strategy}]{\text{Standard Pivoting}} \begin{array}{c} \text{BV} \\ x_2 \\ x_1 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 0 & 1 & \frac{5}{3} & \frac{1}{3} & 0 & 8 \\ 0 & 1 & 0 & -\frac{2}{3} & -\frac{1}{3} & 0 & 4 \\ 0 & 0 & 0 & 2 & -1 & 1 & 46 \\ \hline 1 & 0 & 0 & \frac{14}{3} & \frac{1}{3} & 0 & 44 \end{array}\right] \end{array}$$

The maximum $P = 44$, obtained when $x_1 = 4$ and $x_2 = 8$.

6. Rewrite the constraints:

$$\begin{aligned} -x_1 - \ \ x_2 &\le -11 \\ -2x_1 - 3x_2 &\le -24 \\ x_1 + 3x_2 &\le 18 \\ x_1 \ge 0,\ x_2 &\ge 0 \end{aligned}$$

Introduce slack variables:

$$\begin{aligned} -x_1 - \ \ x_2 + s_1 \qquad\qquad &= -11 \\ -2x_1 - 3x_2 \qquad + s_2 \qquad &= -24 \\ x_1 + 3x_2 \qquad\qquad + s_3 &= 18 \\ x_1 \ge 0,\ x_2 \ge 0\ s_1 \ge 0,\ s_2 \ge 0,\ s_3 &\ge 0 \end{aligned}$$

We now set up the initial tableau and use the alternate pivoting method as long as there are negative entries in the RHS. When all the entries in the RHS are positive, we use the standard way of choosing a pivot.

$$\begin{array}{c} \text{BV} \\ s_1 \\ s_2 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & -1 & -1 & 1 & 0 & 0 & -11 \\ 0 & \boxed{-2} & -3 & 0 & 1 & 0 & -24 \\ 0 & 1 & 3 & 0 & 0 & 1 & 18 \\ \hline 1 & -5 & -2 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \xrightarrow[\substack{R_2 = -\frac{1}{2} r_2 \\ R_1 = R_2 + r_1 \\ R_3 = -R_2 + r_3 \\ R_4 = 5R_2 + r_4}]{\text{Alternative Pivoting Strategy}} \begin{array}{c} \text{BV} \\ s_1 \\ x_1 \\ s_3 \\ P \end{array} \begin{array}{c} \begin{array}{cccccc|c} P & x_1 & x_2 & s_1 & s_2 & s_3 & \text{RHS} \end{array} \\ \left[\begin{array}{cccccc|c} 0 & 0 & 0.5 & 1 & -0.5 & 0 & 1 \\ 0 & 1 & 1.5 & 0 & -0.5 & 0 & 12 \\ 0 & 0 & 1.5 & 0 & 0.5 & 1 & 6 \\ \hline 1 & 0 & 5.5 & 0 & -2.5 & 0 & 60 \end{array}\right] \end{array}$$

Standard Pivoting →
$R_3 = 2r_3$
$R_1 = 0.5R_3 + r_1$
$R_2 = 0.5R_3 + r_2$
$R_4 = 2.5R_3 + r_4$

BV	P	x_1	x_2	s_1	s_2	s_3	RHS
s_1	0	0	2	1	0	1	7
x_1	0	1	3	0	0	1	18
s_2	0	0	3	0	1	2	12
P	1	0	13	0	0	5	90

The maximum $P = 90$, obtained when $x_1 = 18$ and $x_2 = 0$.

12. Maximize $P = -z = -2x_1 - x_2 - x_3$.

Rewrite the constraints:

$$
\begin{aligned}
3x_1 - x_2 - 4x_3 &\le -12 \\
-x_1 - 3x_2 - 2x_3 &\le -10 \\
x_1 - x_2 + x_3 &\le 8 \\
x_1 \ge 0,\ x_2 \ge 0,\ x_3 &\ge 0
\end{aligned}
$$

Introduce slack variables:

$$
\begin{aligned}
3x_1 - x_2 - 4x_3 + s_1 &= -12 \\
-x_1 - 3x_2 - 2x_3 + s_2 &= -10 \\
x_1 - x_2 + x_3 + s_3 &= 8 \\
x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ s_1 \ge 0,\ s_2 \ge 0,\ s_3 &\ge 0
\end{aligned}
$$

We then set up the initial tableau and use the Alternative Pivoting Strategy as long as there are negative entries in the RHS. When all the entries in the RHS are positive, we use the standard way of choosing a pivot.

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
s_1	0	3	[−1]	−4	1	0	0	−12
s_2	0	−1	−3	−2	0	1	0	−10
s_3	0	1	−1	1	0	0	1	8
P	1	2	1	1	0	0	0	0

Alternative Pivoting Strategy →

BV	P	x_1	x_2	x_3	s	s_2	s_3	RHS
x_2	0	−3	1	4	−1	0	0	12
s_2	0	−10	0	[10]	−3	1	0	26
s_3	0	−2	0	5	−1	0	1	20
P	1	5	0	−3	1	0	0	−12

Standard Pivoting Strategy →

BV	P	x_1	x_2	x_3	s_1	s_2	s_3	RHS
x_2	0	1	1	0	$\frac{1}{5}$	$-\frac{2}{5}$	0	$\frac{8}{5}$
x_3	0	−1	0	1	$-\frac{3}{10}$	$\frac{1}{10}$	0	$\frac{13}{5}$
s_3	0	3	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	7
P	1	2	0	0	$\frac{1}{10}$	$\frac{3}{10}$	0	$-\frac{21}{5}$

Maximum $P = -\dfrac{21}{5}$, so minimum $z = \dfrac{21}{5}$, and is obtained when $x_1 = 0$, $x_2 = \dfrac{8}{5}$, and $x_3 = \dfrac{13}{5}$.