

- 34.** Five different boxes can be stacked  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$  ways.
- 35.** There are  $5 \cdot 4 \cdot 3 = 60$  different three-letter codes that can be formed from the 5 letters  $A, B, C, D,$  and  $E$  if no letter can be used more than once.

This is the same as finding  $P(5, 3) = 60$ .

- 49.** Since the 13<sup>th</sup> digit is determined by the other 12, there is no choice in picking it. The other 12 places are free to vary and repetitions are allowed. So there are  $10^{12}$  possible ISBN numbers.
- 52.** This is a permutation in which 5 different days are selected from a possible 365 days without a repetition. It is given by

$$P(365, 5) = \frac{365!}{(365-5)!} = \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360!}{360!} = 365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 = 6,302,555,019,000$$

So, 5 people can each have a different birthday in more than 6.3 trillion different ways.

- 53.** (a) SUNDAY has 6 letters, and none of them are repeated. They can be arranged  $P(6, 6) = 6! = 720$  different ways.
- (b) If the letter S must come first, then we are really only arranging 5 letters. This can be done  $P(5, 5) = 5! = 120$  different ways.
- (c) If the letter S must come first and the letter Y must come last, then only four letters are being arranged. It is the permutation of 4 objects which can be done  $P(4, 4) = 4! = 24$  ways.

- 54.** There are 5 French books and 5 Spanish books to be arranged.

- (a) If books of the same language must be together with French on the left and Spanish on the right, we have two tasks. The first is to arrange the 5 French books. This can be done  $P(5, 5)$  ways. The second task is to arrange the 5 Spanish books, which also can be done  $P(5, 5)$  ways. By the Multiplication Principle the ten books can be placed on the shelf  $P(5, 5) \cdot P(5, 5) = 5! \cdot 5! = (5!)^2 = 14,400$  ways.
- (b) Alternating the French and Spanish books in the grouping, starting with a French book, still can be done in 14,400 ways, since there are 5 choices for the first French book, 5 choices for the first Spanish book, 4 choices for the second French book, 4 choices for the second Spanish book, and so on. Using the Multiplication Principle, we get  $5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 5! \cdot 5! = 14,400$  ways.