6.5 (page 390)

14. The combinations of 5 objects *a*, *b*, *c*, *d*, and *e* taken 2 at a time are:

ab, *ac*, *ad*, *ae*, *bc*, *bd*, *be*, *cd*, *ce*, *de*

$$C(5,2)=\frac{5!}{(5-2)!\ \ 2!}=\frac{5\cdot 4\cdot 3!}{3!\ \ 2!}=\frac{5\cdot 4}{2\cdot 1}=10$$

17. A committee of 4 students chosen from a pool of 7 students can be formed $C(7, 4) = 35$ ways.

$$C(7,\ 4)=\frac{7!}{(7-4)!\ 4!}=\frac{7\cdot 6\cdot 5\cdot 4!}{3!\ 4!}=\frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1}=35$$

24. A bit is either a 0 or a 1. An 8-bit string is a list of 8 digits, all of which are either 0s or 1s. To determine how many 8-bit strings contain exactly two 1s we need to select 2 positions to place the 1s. This can be done

$$C(8,\ 2)=\frac{8!}{-}=\frac{8\cdot 7\cdot 6!}{}=\frac{8\cdot 7}{\cdot}=28\ \text{ ways.}$$

27. In the word ECONOMICS there are 9 letters, but they are not all distinct. There are 2 C's, 2 O's, and one each of the remaining 5 letters. We want the number of permutations of 9 objects, not all of which are distinct.

The number of 9-letter words that can be formed is given by

$$\frac{9!}{2!\ 2!\ 1!\ 1!\ 1!\ 1!\ 1!}=90,720$$

31. This problem consists of two tasks: selecting the boys for the committee which can be done C(4, 2) ways and selecting the girls for the committee which can be done $C(8, 3)$ ways. Then by the Multiplication Principle, we find that the committee can be formed in

$$C(4,\ 2)\cdot C(8,\ 3)=\frac{4!}{(4-2)!\ 2!}\cdot\frac{8!}{(8-3)!\ 3!}=\frac{4\cdot 3\cdot 2!}{2\cdot 1\cdot 2!}\cdot\frac{8\cdot 7\cdot 6\cdot 5!}{5!\cdot 3\cdot 2\cdot 1}=6\cdot 56=336\ \text{ ways.}$$

33. Once on a team the children are no longer distinct, so placing people on teams is much like forming words in which all the letters are not distinct.

The 12 children can be placed on 3 teams, a first having 3 players, a second having 5 players and a third having 4 players in

$$\frac{12!}{3!\cdot 5!\cdot 4!}=\frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5!}{3\cdot 2\cdot 1\cdot 5!\cdot 4\cdot 3\cdot 2\cdot 1}=1\cdot 11\cdot 10\cdot 9\cdot 4\cdot 7\cdot 1=27,720\ \text{ ways}$$

39. We want to select 8 accounts from 58 accounts without regard to the order of selection. This can be done $C(58, 8) = 1,916,797,311$ different ways.

40. Since 5 of the accounts contain errors we separate the 58 accounts into two subsets: 53 that are in good order and 5 that have errors.

(a) We want to choose 7 accounts from the 53 good accounts and 1 account from the 5 faulty accounts. This can be done

$$C(53, 7) \cdot C(5, 1) = \frac{53!}{46!7!} \cdot 5 = 154{,}143{,}080 \cdot 5 = 770{,}715{,}400 \text{ ways.}$$

(b) Exactly 2 have errors: $C(53, 6) \cdot C(5, 2) = 229{,}574{,}800$

(c) Exactly 3 have errors: $C(53, 5) \cdot C(5, 3) = 28{,}696{,}850$

(d) Exactly 4 have errors: $C(53, 4) \cdot C(5, 4) = 1{,}464{,}125$

(e) Exactly 5 have errors: $C(53, 3) \cdot C(5, 5) = 23{,}426$

(f) The number of sample that have at least one account with an error is the sum of the number of samples with one account with an error, the number of samples with two accounts with errors, the number of samples with two accounts with errors and so on. There are

$$770{,}715{,}400 + 229{,}574{,}800 + 28{,}696{,}850 + 1{,}464{,}125 + 23{,}426 = 1{,}030{,}474{,}601$$

samples that contain at least one account that has an error.

(g) The number of samples that contain no errors is $C(53, 8) \cdot C(5, 0) = 886{,}322{,}710$.

47. Since we do not care whether the sample includes smokers or non-smokers, we add the two groups together. Then we select the sample. This can be done $C(55, 8)$ ways.

$$C(55,\ 8) = \frac{55!}{(55-8)!\ 8!} = \frac{55!}{47!\ 8!} = 1{,}217{,}566{,}350 \text{ ways.}$$

48. There are two tasks to be done here. The first is to select the trucks which can be done $C(8, 4)$ ways; the second is to select the drivers which can be done $C(6, 4)$ ways. By the Multiplication Principle there are

$$C(8,\ 4)\cdot C(6,\ 4) = \frac{8!}{(8-4)!\ 4!}\cdot\frac{6!}{(6-4)!\ 4!} = \frac{8!}{4!\ 4!}\cdot\frac{6!}{2!\ 4!} = 70\cdot 15 = 1050$$

ways to meet the requests.