6.6 (page 399)

10. $\binom{100}{98} = \frac{100!}{(100-98)!\,98!} = \frac{100!}{2!\,98!}$

$= \frac{100 \cdot 99}{2} = 4950$

11. $\binom{1000}{1000} = \frac{1000!}{(1000-1000)!\,1000!} = \frac{1}{0!} = 1$

19. $(x+3y)^3 = \binom{3}{0}x^3(3y)^0 + \binom{3}{1}x^2(3y)^1 + \binom{3}{2}x^1(3y)^2 + \binom{3}{3}x^0(3y)^3$

$= x^3 + 3x^2 \cdot 3y + 3x \cdot 9y^2 + 27y^3 \quad = x^3 + 9x^2y + 27xy^2 + 27y^3$

21. $(2x-y)^4 = \binom{4}{0}(2x)^4(-y)^0 + \binom{4}{1}(2x)^3(-y)^1 + \binom{4}{2}(2x)^2(-y)^2 + \binom{4}{3}(2x)^1(-y)^3 + \binom{4}{4}(2x)^0(-y)^4$

$= 16x^4 - 4 \cdot 8x^3y + 6 \cdot 4x^2y^2 - 4 \cdot 2xy^3 + y^4$

$= 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$

24.

$\binom{8}{0}x^8 + \binom{8}{1}x^7y + \binom{8}{2}x^6y^2 + \binom{8}{3}x^5y^3 + \binom{8}{4}x^4y^4 + \binom{8}{5}x^3y^5 + \binom{8}{6}x^2y^6 + \binom{8}{7}xy^7 + \binom{8}{8}y^8$

The coefficient of x^2y^6 is $\binom{8}{6} = 28$.

26.

$(x+2)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4 \cdot 2 + \binom{5}{2}x^3 \cdot 2^2 + \binom{5}{3}x^2 \cdot 2^3 + \binom{5}{4}x \cdot 2^4 + \binom{5}{5}2^5$

The coefficient of x^3 is $\binom{5}{2} \cdot 2^2 = 40$.

A set with 50 elements has 2^{50} = 1,125,899,907,000,000 different subsets.

28.

34. To show that $\binom{8}{5} = \binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4}$, we repeatedly use the identity

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

So,

$$\binom{8}{5} = \binom{7}{5} + \binom{7}{4}$$
$$= \left[\binom{6}{5} + \binom{6}{4}\right] + \binom{7}{4}$$
$$= \left[\binom{5}{5} + \binom{5}{4}\right] + \binom{6}{4} + \binom{7}{4}$$

Now since $\binom{5}{5} = \binom{4}{4} = 1$, we substitute $\binom{4}{4}$ for $\binom{5}{5}$ in the line above and pr

$$\binom{8}{5} = \binom{4}{4} + \binom{5}{4} + \binom{6}{4} + \binom{7}{4}$$