5. The number of elements in the sample space S is found by using the Multiplication Principle. Since the coin is tossed 5 times, there are

$$n(S) = 2^5 = 32 \text{ elements.}$$

(a) Define E as the event, "Exactly 3 heads appear."

$$P(E) = \frac{n(E)}{n(S)} = \frac{C(5,\ 3)}{32} = \frac{10}{32} = \frac{5}{16}$$

(b) Define F as the event, "No heads appear."

$$P(F) = \frac{n(F)}{n(S)} = \frac{C(5,\ 0)}{32} = \frac{1}{32}$$

7. The number of elements in the sample space S is found by using the Multiplication Principle. Since the dice are thrown 3 times,

$$n(S) = 36^3 = 46{,}656$$

(a) Define E as the event, "The sum of 7 appears 3 times."
When two dice are thrown, a sum of 7 can appear 6 ways, $\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$. Using the Multiplication Principle we find that on 3 throws a sum of 7 can appear 6^3 ways.

$$P(E) = \frac{n(E)}{n(S)} = \frac{6^3}{36^3} = \frac{1}{6^3} = \frac{1}{216} \approx 0.005$$

(b) Define F as the event, "The sum of 7 or 11 appears at least twice."
When 2 dice are thrown a sum of 7 can appear 6 ways and a sum of 11 can appear 2 ways. Using the addition rule, the sum of 7 or 11 can appear 8 ways.

Event F is equivalent to the event: Exactly two throws result in a 7 or 11, or all three throws result in a 7 or 11. These are mutually exclusive, so we are interested in finding

$$\mathrm{P}(F) = P(7 \text{ or } 11 \text{ appears twice}) + P(7 \text{ or } 11 \text{ appear 3 times})$$

Using the Multiplication Principle 7 or 11 can appear 3 times in $8^3 = 512$ ways.

7 or 11 can appear 2 times in $8^2 = 64$ ways, and the third number can appear $36 - 8 = 28$ ways. By the Multiplication Principle 2 throws of 7 or 11 and 1 throw of another number can appear $8^2 \cdot 28 = 1792$ ways. However, we still need to choose which of the two throws will result in the 7 or 11. We choose 2 out of 3 tries $C(3, 2) = 3$ ways.

$$P(F) = \frac{n(F)}{n(S)} = \frac{C(3,\ 2)\cdot 8^2 \cdot 28}{36^3} + \frac{8^3}{36^3}$$

$$= \frac{5376}{36^3} + \frac{512}{36^3} = \frac{5888}{46656} = 0.126$$

9. The number of elements in the sample space, S, is found by using the Multiplication Principle. Since there are 7 digits in the phone number, and there are no restrictions,

$$n(S) = 10^7$$

Define event E: A phone number has one or more repeated digits.

It is easier to do this problem looking at the complement of E. $\overline{E}$: A phone number has no repeated digits. $n(\overline{E}) = P(10, 7)$. So we get

$$\begin{aligned} P(E) &= 1 - P(\overline{E}) \\ &= 1 - \frac{n(\overline{E})}{n(S)} \\ &= 1 - \frac{P(10,\ 7)}{10^7} = 0.940 \end{aligned}$$

11. The number of elements in the sample space, S, is found by using the Multiplication Principle. Since there are 26 letters in the alphabet and we are going to select 5, allowing repetitions

$$n(S) = 26^5$$

Define event E: No letters are repeated. $n(E) = C(26, 5)$

$$P(E) = \frac{n(E)}{n(S)} = \frac{C(26,\ 5)}{26^5} = 0.0055$$

13. The number of elements in the sample space, S, is found by using the Multiplication Principle. Since there are 12 months in the year, and we are going to select 3,

$$n(S) = 12^3 = 1728$$

Define event E: At least 2 were born in the same month. It is easier to do this problem by looking at the complement of E,

$\overline{E}$: No two people were born in the same month.

$$n(\overline{E}) = P(12, 3) = 12 \cdot 11 \cdot 10 = 1320$$

The probability that at least 2 were born in the same month is

$$\begin{aligned} P(E) &= 1 - P(\overline{E}) \\ &= 1 - \frac{n(\overline{E})}{n(S)} \\ &= 1 - \frac{1320}{1728} = 0.236 \end{aligned}$$

15. The number of elements in the sample space, S, is found by using the Multiplication Principle. Since there are 365 days in a year, and 100 senators, $n(S) = 365^{100}$.

Define event E: At least 2 senators have the same birthday. It is easier to do this problem by looking at the complement of E,

$\overline{E}$: No two senators have the same birthday.

$$n(\overline{E}) = P(365, 100) = 365 \cdot 364 \cdot 363 \cdot \ldots \cdot 266$$

The probability that at least 2 senators have the same birthday is

$$\begin{aligned} P(E) &= 1 - P(\overline{E}) \\ &= 1 - \frac{n(\overline{E})}{n(S)} = 1 - \frac{P(365,\ 100)}{365^{100}} = 0.99999 \approx 1 \end{aligned}$$

21. The experiment consists of choosing 5 accounts from a list of 35 accounts. The accounts can be partitioned into two groups: 32 that are correct and 3 that contain errors. The number of elements in the sample space is $C(35, 5) = 324{,}632$.

(a) The probability of choosing 5 accounts without errors:

$$\frac{C(32,5)\cdot C(3,0)}{C(35,5)}=\frac{\frac{32!}{5!\cdot 27!}\cdot 1}{\frac{35!}{5!\cdot 30!}}=\frac{116}{187}\approx 0.620$$

(b) The probability of choosing exactly 1 account with errors is

$$\frac{C(32,4)\cdot C(3,1)}{C(35,5)}=\frac{\frac{32!}{4!\cdot 28!}\cdot 3}{324{,}632}=\frac{435}{1309}\approx 0.332$$

(c) The probability of choosing at least 2 accounts containing errors is easiest done by using the complement. The probability of choosing at least 2 accounts containing errors is 1 minus the probability of choosing fewer than 2 accounts with errors.

1 – [probability of choosing 5 accounts with no errors + probability of choosing 4 accounts with no errors] = 1 – 0.620 – 0.332 = 0.048

24. Each pin number is two letters followed by three numbers. Repetition is allowed and order is important.

(a) There are $26^2 \cdot 10^3 = 676{,}000$ different pin numbers.

(b) There are $26^2 \cdot 1 = 676$ numbers that end in 000. The probability of a number ending in 000 is

$$\frac{n(\text{end in }000)}{n(\text{pin numbers})}=\frac{676}{676{,}000}=\frac{1}{1000}=0.001$$

(c) There are $1 \cdot 10^3 = 1000$ pin numbers that begin with AA. The probability of a number beginning with AA is

$$\frac{n(\text{begin with AA})}{n(\text{pin numbers})}=\frac{1000}{676{,}000}=\frac{1}{676}\approx 0.001$$

(d) There are $26 \cdot 10 = 260$ numbers that begin with A and end with 00. The probability a number begins with A and ending with 00 is

$$\frac{n(\text{begins with A and end with }00)}{n(\text{pin numbers})}=\frac{260}{676{,}000}=\frac{1}{2600}\approx 0.0038$$

(e) There are $26 \cdot P(10, 3) = 18{,}720$ pin numbers that have a repeated letter and different numbers. The probability a number has a repeated letter and different numbers is

$$\frac{n(\text{letter repeated; numbers different})}{n(\text{pin numbers})}=\frac{18{,}720}{676{,}000}=\frac{9}{325}\approx 0.028$$

34. The experiment consists of choosing 2 cans of soda from 12 cans. The sample space is all combinations of 2 that can be chosen from 12. $n(S) = C(12, 2) = 66$

(a) Define event E as both cans are regular soda. $n(E) = C(3, 2) = 3$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{66} = \frac{1}{22} \approx 0.045$$

(b) Define event F as both cans contain diet soda; $n(F) = C(9, 2) = 36$

$$P(F) = \frac{n(F)}{n(S)} = \frac{36}{66} = \frac{6}{11} \approx 0.545$$

(c) Define event G as one can is regular and one can is diet soda; $n(G) = C(9, 1) \cdot C(3, 1) = 27$

$$P(G) = \frac{n(F)}{n(S)} = \frac{27}{66} = \frac{9}{22} \approx 0.409$$