7.4 (page 448)

16. $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$

$$P(E \cap F) = P(F) \cdot P(E \mid F) = (0.38)(0.46) = 0.1748$$

18. (a) $P(F \mid E) = \dfrac{P(E \cap F)}{P(E)}$

$$P(E) = \frac{P(E \cap F)}{P(F \mid E)} = \frac{0.1}{0.125} = \frac{100}{125} = \frac{4}{5}$$

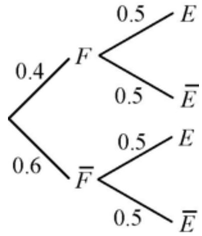
25. $P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$$= 0.5 + 0.4 - 0.8 = 0.1$$

26. $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{0.1}{0.4} = \dfrac{1}{4}$

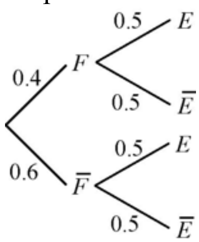
27. $P(F \mid E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{0.1}{0.5} = \dfrac{1}{5} = 0.2$

28. A tree diagram helps to see this problem.



$$P(\overline{E} \mid F) = \frac{P(\overline{E} \cap F)}{P(F)} = \frac{0.3}{0.4} = \frac{3}{4}$$

29. A tree diagram helps to see this problem.



$$P(E \mid \overline{F}) = \frac{P(E \cap \overline{F})}{P(\overline{F})} = \frac{0.4}{0.6} = \frac{2}{3} \approx 0.667$$

30. From De Morgan's properties (Chapter 6), we know

$$\overline{E} \cap \overline{F} = \overline{E \cup F}.$$

So, $\quad P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F})$

and $P(\overline{E \cup F}) = 1 - P(E \cup F) = 1 - 0.8 = 0.2$

$$P(\overline{E} \mid \overline{F}) = \frac{P(\overline{E} \cap \overline{F})}{P(\overline{F})} = \frac{0.2}{0.6} = \frac{1}{3} \approx 0.333$$

32. Define event E: A family has more than 2 children.
Define event F: A family has at least 1 child.

$$E \cap F = E;\ P(E \cap F) = 0.20 + 0.16 + 0.08 + 0.06 = 0.50$$
$$P(F) = 0.30 + 0.20 + 0.16 + 0.08 + 0.06 = 0.80$$

The probability that a family has more than 2 children if it is known that it has at least 1 child is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{0.5}{0.8} = \frac{5}{8}$$

34. The experiment consists of drawing two cards without replacement from a deck of 52 cards.

(a) Define event E: "The second card is a queen." The event can be considered the union of two mutually exclusive events:

A: the first card is a queen and the second card is a queen;
B: the first card is not a queen and the second card is a queen.

$$P(E) = P(A) + P(B) = \frac{4}{52}\cdot\frac{3}{51} + \frac{48}{52}\cdot\frac{4}{51} = \frac{1}{13}$$

(b) The probability that both cards are queens is

$$P(A) = \frac{4}{52}\cdot\frac{3}{51} = \frac{1}{(13)(17)} = \frac{1}{221}$$

35. The experiment consists of drawing two balls without replacement from an box containing 8 balls. Define event E: A white ball and a yellow are drawn.

The probability of choosing a white and a yellow ball without replacement can be considered as the union of two mutually exclusive events.

$$P(E) = P(\text{W on first}) \cdot P(\text{Y} \mid \text{W on first}) + P(\text{Y on first}) \cdot P(\text{W} \mid \text{Y on first})$$
$$= \frac{3}{6}\cdot\frac{1}{5} + \frac{1}{6}\cdot\frac{3}{5} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

37. The experiment consists of drawing a card from a deck of 52 cards; $n(S) = 52$.

(a) Define event E to be "A red ace is drawn." $n(E) = 2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(b) Define event F to be "An ace is drawn." $n(F) = 4$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{52}}{\frac{4}{52}} = \frac{1}{2}$$

(c) Define event G to be "A red card is picked." $n(G) = 26$

$$P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{2}{52}}{\frac{26}{52}} = \frac{2}{26} = \frac{1}{13}$$

65. (a) $P(M)=\dfrac{1448}{2018}=\dfrac{724}{1009}\approx 0.7175$

(b) $P(A)=\dfrac{666}{2018}=\dfrac{333}{1009}\approx 0.3300$

(c) $P(F\cap B)=\dfrac{144}{2018}=\dfrac{72}{1009}\approx 0.0714$

(d) $P(F\mid E)=\dfrac{P(F\cap E)}{P(E)}=\dfrac{102}{526}=\dfrac{51}{263}\approx 0.194$

(e) $P(A\mid M)=\dfrac{P(A\cap M)}{P(M)}=\dfrac{342}{1448}=0.2362$

(f) $P(F\mid A\cup E)=\dfrac{P\left[F\cap(A\cup E)\right]}{P(A\cup E)}=\dfrac{P(F\cap A)+P(F\cap E)}{P(A)+P(E)}=\dfrac{324+102}{666+526}=\dfrac{426}{1192}=\dfrac{213}{593}\approx 0.357$

(g) $P(M\cap\overline{B})=P\left[M\cap(A\cup E)\right]=P(M\cap A)+P(M\cap E)=\dfrac{342}{2018}+\dfrac{424}{2016}=\dfrac{766}{2018}=\dfrac{383}{1009}\approx 0.380$

(h) $P(F\mid\overline{E})=P(F\mid A\cup B)=\dfrac{P\left[F\cap(A\cup B)\right]}{P(A\cup B)}=\dfrac{P(F\cap A)+P(F\cap B)}{P(A)+P(B)}=\dfrac{324+144}{666+826}=\dfrac{468}{1492}=\dfrac{117}{373}$