7.5 (page 458)

4. $P(F \cap E) = P(E) \cdot P(F)$

$$P(F) = \frac{P(F \cap E)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

6. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= P(E) + P(F) - P(E)P(F)$$

$$= P(E)\left(1 - P(F)\right) + P(F)$$

$$P(E) = \frac{P(E \cup F) - P(F)}{1 - P(F)}$$

$$= \frac{0.6 - 0.3}{1 - 0.3} = \frac{0.3}{0.7} = \frac{3}{7}$$

9. (a) $P(E \mid F) = P(E) = 0.2$

(c) $P(E \cap F) = P(E) \cdot P(F)$

$= (0.2)\,(0.4)$

$= 0.08$

12. $P(E_1 \cap E_2 \cap E_3 \cap E_4)$

$= P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot P(E_4)$

$= (0.6)(0.3)(0.5)(0.4) = 0.036$

14. $P(E \cap F) = P(E) + P(F) - P(E \cup F)$

$= 0.4 + 0.6 - 0.7 = 0.3$

$$P(E \mid F) = \frac{P(F \cap E)}{P(F)} = \frac{0.3}{0.6} = \frac{1}{2}$$

$P(E) \cdot P(F) = (0.4)(0.6) = 0.24 \neq P(E \cap F)$

E and F are not independent.

16. (a) C: A club is drawn first.

$$P(\mathrm{C}) = \frac{13}{52} = \frac{1}{4}$$

(b) C: A club is drawn first.
H: A heart is drawn second.
Since the events are independent,

$$P(H \mid C) = P(H) = \frac{13}{52} = \frac{1}{4}$$

(c) C: A club is drawn first.
H: A heart is drawn second.

$P(C \cap H) = P(C) \cdot P(H)$

$$= \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$$

(d) A: The first card is an ace.

$$P(A) = \frac{4}{52} = \frac{1}{13}$$

(e) A: The first card is an ace.
K: A king is drawn second.
Since the events are independent,

$$P(K \mid A) = P(K) = \frac{4}{52} = \frac{1}{13}$$

(f) A: The first card is an ace.
K: A king is drawn second.

$$P(A \cap K) = P(A) \cdot P(K) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13^2} = \frac{1}{169}$$

18. $n(S) = 100$; $n(E) = 52 + 18 = 70$; $n(\overline{E}) = 8 + 22 = 30$; $n(F) = 52 + 8 = 60$; $n(\overline{F}) = 18 + 22 = 40$

$$P(E) = \frac{70}{100} = 0.7 \qquad P(\overline{E}) = \frac{30}{100} = 0.3 \qquad P(F) = \frac{60}{100} = 0.6 \qquad P(\overline{F}) = \frac{40}{100} = 0.4$$

(a) $P(E \cap F) = \frac{52}{100} = 0.52$

$P(E)P(F) = 0.7 \cdot 0.6 = 0.42$

$P(E)P(F) \neq P(E \cap F)$

The events are not independent.

(b) $P(\overline{E} \cap \overline{F}) = \frac{22}{100} = 0.22$

$P(\overline{E})P(\overline{F}) = 0.3 \cdot 0.4 = 0.12$

$P(\overline{E})P(\overline{F}) \neq P(\overline{E} \cap \overline{F})$

The events are not independent.

(c) $P(E \cap \overline{F}) = \frac{18}{100} = 0.18$

$P(E)P(\overline{F}) = 0.7 \cdot 0.4 = 0.28$

$P(E)P(\overline{F}) \neq P(E \cap \overline{F})$

The events are not independent.

19. Let H denote a child with heart disease, and $\overline{H}$ denote a child with no heart disease. The sample space is the set of all possible outcomes. The couple has two children. $S = \{HH, H\overline{H}, \overline{H}H, \overline{H}\overline{H}\}$

$$P(H) = \frac{3}{4} \qquad P(\overline{H}) = \frac{1}{4}$$

(a) $P(HH) = P(H \cap H)$

$= P(H) \cdot P(H)$

$= \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

(b) $P(\overline{H}\overline{H}) = P(\overline{H} \cap \overline{H})$

$= P(\overline{H}) \cdot P(\overline{H})$

$= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

(c) $P(H\overline{H} \cup \overline{H}H) = P(H \cap \overline{H}) + P(\overline{H} \cap H) = P(H) \cdot P(\overline{H}) + P(\overline{H}) \cdot P(H) = \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} = \frac{6}{16} = \frac{3}{8}$