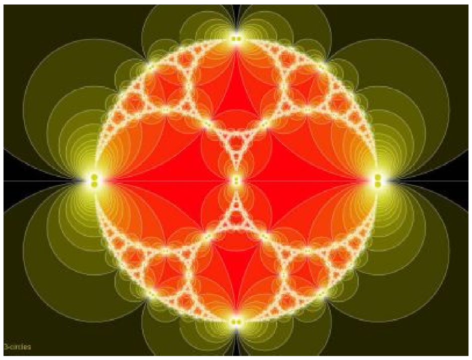
Topic Course Proposal
Jerzy Kocik

Symmetry and Geometry
applications of group theory



> The mathematical sciences particularly exhibit order, symmetry, and grace; and these are the greatest forms of beauty.
>
> from Aristotle
> Metaphysics

Symmetry is a notion that expresses our intuitive grasp of beauty, harmony, balance and aesthetically gratifying proportionality. The mathematical study of symmetry has been formalized in terms of *group theory*, which finds application in practically every domain of mathematics and the sciences. It is a powerful key that opens doors to various hidden structures.

In this course we will study groups and associated algebraic structures with emphasis on understanding low-dimensional examples and their matrix representations. Special attention will be given to rotations, the Moebius group, SL(2,**Z**), SL(2,**C**), SU(2) hyperbolic rotations SO(n,1), Apollonian group. Various relationships among them will be analyzed and exemplified. Topics will include algebraic structures like quaternions, octonions, Lie algebras, Clifford algebras, spin structure, Pythagorean triples, integer sequences, geometry of Descartes configuration in $\mathbf{R}^n$, projective geometry, elements of fractal geometry.

Applications in geometry will bring an occasion to build and verify one's mathematical intuition. A certain fractal arrangement of circles (or higher-dimensional spheres) known as *integral Apollonian circle packing*, will be used as a vehicle to wander through these topics. It conceals a multitude of far-reaching connections within mathematics: from geometry to group theory to number theory, many of them discovered very recently. The course will include reading of recent relevant papers (see below).

Audience: The topics to be covered are of universal importance and go beyond the specific applications. Students of diverse interests are invited to participate in this class.

Prerequisite: Basic linear algebra.

Text: Excerpts from a number of books will be made available. Also, lots of handouts will be freely distributed.

Additional information: http://www.math.siu.edu/Kocik/symmetry/symmetry.html

Initial guides to the literature

David Mumford , Caroline Series and David Wright: **Indra's Pearls: The Vision of Felix Klein.** Cambridge University Press in 2002.

B. B. Mandelbrot, **The Fractal Geometry of Nature**, Freeman: New York, 1982.

John Stillwell: **Naïve Lie theory**, Springer 2008 (a very simple and friendly introduction to Lie groups and algebras)

B. Söderberg, **Apollonian tiling, the Lorentz group, and regular trees**. *Phys. Rev. A* 46 (1992), No. 4, 1859–1866.

Alan F. Beardon, **The Geometry of Discrete Groups**, Springer-Verlag: New York 1983.

Mark A. Armstrong: **Groups and symmetry**, 3rd printing, Springer-Verlag 1997 (a nice introduction to the basic concepts)

Tristan Needham, **Visual Complex Analysis**, Oxford University Press, 1999.

Kristopher Tapp, **Matrix Groups for Undergraduates (Student Mathematical Library),** AMS, 2005

O.L. Weaver, David H. Sattinger: **Lie Groups and Algebras With Applications to Physics, Geometry, and Mechanics** (Applied Mathematical Sciences, Vol 61) Springer 1986. 215 pages

D. Pedoe, **On Circles. A Mathematical View**. Enlarged edition by Dover (1979).

Resources on **Symmetry**, **Group Theory**, and related subjects: www.theophys.kth.se/mathphys/SYM/sym_link.html

R. L. Graham, J. C. Lagarias, C. L. Mallows, A.Wilks and C. Yan, Apollonian circle packings: **geometry and group theory** I. Apollonian group, *Discrete and Computational Geometry*, 34: 547-585(2005)

———, Apollonian circle packings: **geometry and group theory II.** Super-Apollonian group and integral packings, *Discrete and Computational Geometry*, 35: 1-36(2006)

———, Apollonian circle packings: **geometry and group theory III.** Higher dimensions, *Discrete and Computational Geometry*, 35: 37-72(2006)

———, Apollonian circle packings: **number theory**, *Number Theory* 100 (2003), 1–45.

J. B. Wilker, **Inversive Geometry**, in: The Geometric Vein, (C. Davis, B. Grünbaum, F. A. Sherk, Eds.), Springer-Verlag: New York 1981, pp. 379–442.

Other interesting resources will be given at a later time.