Exam 1 on Friday, Feb. 11 covers sections 6.1,6.2,6.3,6.4, and 6.5. You are allowed a NON-GRAPHICAL calculator (no TI 82, 83 or 85) but NO NOTES.

A **set** consists of distinct elements enclosed by *braces*, eg $\{1, 5, 7\}$. The *empty set* $\emptyset$ is the set that contains no elements. $A = B$ iff A and B have the same elements.

A is a subset of B, $A \subseteq B$, if every element in A is in B.

A is a proper subset of B, $A \subset B$, if $A \subseteq B$ but there is at least one element in B that is not in A.

The *universal set* U is the set of all elements under consideration (the biggest set).

The **union** of A with $B = A \cup B$ is the set of all elements in A, in B or in both.

The **intersection** of A with $B = A \cap B$ is the set of all elements in A and B.

If $A \cap B = \emptyset$, then A and B are *disjoint sets.*

The *complement* of A is $\overline{A}$, the set of elements in U but not in A.

A) **Common Problem:** Given U, A, B, and C, find things like $\overline{A}$, $A \cup B$, $A \cap B$, whether $A \subset B$ or $A \subseteq B$, whether A and C are disjoint. See Q1 3) and HW1 12), 15), 20).

B) **Common Problem:** Given 3 of the 4 numbers in the counting formula $c(A \cup B) = c(A) + c(B) - c(A \cap B)$, find the 4th. See Q1 1), 2), HW1 7), 10) and HW4 29 rev.

C) **Common Problem:** Given a Venn diagram with counts in each region, use the regions to find counts of subsets of interest. See HW1 16), 18), 20).

1) **Common Final Problem:** You are given a story problem with 3 sets A, B, and C. From the story problem, from *top to bottom list the counts* of the biggest set U, then of the 3 single sets, then of the 3 pairwise intersections, then the 3 way intersection: $c(U), c(A), c(B), c(C), c(A \cap B), c(A \cap C), c(B \cap C)$ and $c(A \cap B \cap C)$. DRAW A VERTICAL LINE TO THE RIGHT OF THE LIST. Then you will draw a Venn diagram or be given a Venn diagram of the three sets. This Venn diagram divides U into 8 regions of the form $A^* \cap B^* \cap C^*$ where $A^* = A$ if the region is contained in set A and $A^* = \bar{A}$ otherwise. Next *from the bottom* of the counts *work up*, filling in the counts of the eight regions.

(The count for the middle most region is $c(A \cap B \cap C)$. Next fill in the regions corresponding to two way intersections. Each two way intersection is made of two regions, one of which is the middlemost. So $c(B \cap C) - c(A \cap B \cap C)$ is the count for the second region that makes up $B \cap C$. After counts for the three regions corresponding to pairwise intersections are found, fill in the remaining regions corresponding to A, B and C. The count of each of these single sets is the sum of the counts for four regions. For example, $c(C) - c(A \cap \overline{B} \cap C) - c(A \cap B \cap C) - c(\overline{A} \cap B \cap C)$ is the count of the 4th region that makes up region C. Finally subtract from $c(U)$ the 7 counts that have been filled in to get the count for the 8th region $c(\overline{A} \cap \overline{B} \cap \overline{C})$. Use these counts to answer story problem questions. The last count was for *none* of A, B, and C.)

See S04 1, F02 12, S02 7, Q2 1-3, HW1 33, HW2 28, HW4 36

Variant i): if $C(U)$ is not given, you can't find the count for "none." See F03 5.

Variant ii): you are given counts for several 3 way intersections. List the 8 counts $C(U), ..., c(A \cap B \cap C)$ and then add the additional counts for other 3 way intersections. You can immediately put the count for any 3 way intersection on the Venn diagram. Then work from bottom to top to fill in all 8 regions. See F01 5, Q3 1-3, HW2 29

Variant iii) Only 2 sets are given. Fill in counts for 4 regions. See F04 3.

Use diagram to find counts of (eg) "None" = "not any," "exactly one", "all," "total number," "only A" "A but neither B nor C." Note that "or" tends to mean union while "and" tends to mean intersection.

The **multiplication principle** says that if there are n_1 ways to do a first task, n_2 ways to do a 2nd task, ..., and n_k ways to do a kth task, then the number of ways to perform the total act of performing the 1st task, then the second task, ..., then the kth task is $n_1 \cdot n_2 \cdot n_3 \cdots n_k$. (Multiply the k numbers together.)

Techniques for multiplication principle: a) use a slot for each task and write n_i above the ith task. There will be k slots, one for each task. b) use a tree diagram.

2) **Common Final Problem:** Use the multiplication principle to find *how many* ways to perform the tasks. Important special cases include finding the number of license plates or serial numbers. See S04 6, F03 6, S03 4,7, Q2 4,5, Q3 7, HW2 2,5,7,9,15,16,18 HW3 19, 21, 28, 39c, 41.

A special case of the multiplication principle is when $n_1 = n_2 = \cdots = n_k \equiv n$. Then the number of ways to do the k tasks is n^k. This is called a *power*. Note that n is placed above each of the k slots and the *exponent* k is the number of slots. The number of ways to answer k TF questions (2^k) and the number of ways to answer k multiple choice questions with n options (n^k) where $n = 4$ or 5 are typical examples.

Another special case is the *number of r-permutations* of n distinct objects $= P(n, r) = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1) = \frac{n!}{(n-r)!}$. The r-permutation problem has r slots and *order is important*. No object is allowed to be repeated in the arrangement. Typical questions include *how many ways to* "choose r people from n and arrange in a line," "to make r letter words with no letter repeated", "to make 7 digit phone numbers with no digit repeated." Key words include *order, no repeated* and *different.*

3) **Common Final Problem.** A story problem says "how many ways can r *different* objects be *distributed* to n (people or) children in such a way *that no child gets more than one object*?" Solution: $P(n, r)$ ways. See S03 7. Q3 6, HW 3 41.

A special case of permutations is $P(n, n) = n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdots 4 \cdot 3 \cdot 2 \cdot 1 = n \cdot (n-1)! = n \cdot (n-1) \cdot (n-2)! = n \cdot (n-1) \cdot (n-2) \cdot (n-3)! = \cdots$. Typical problems include the number of ways to arrange n books, to arrange the letters in the word CLIPS (5!) etc.

An r-combination is an unordered selection of using r of n distinct objects. The *number of r-combinations* is $C(n, r) = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$. This formula is used in story problems where *order is not important.* Key words include *committees, selecting* (eg 4 people from 10), *choose*, and *unordered.*

4) **Common Final Problem.** Compute $C(n, r) = \binom{n}{r}$, $P(n, r)$, and $n!$ showing all work. YOUR FINAL ANSWER SHOULD ALWAYS BE AN INTEGER. F04 1, S04 2, F03 1, F02 2, S02 2, F01 1, Q3 4, HW3 2,3,7, HW4 37, 45, 52, 55.

5) **Common Final Problem.** Often the multiplication principle will be combined with combinations, permutations, powers and factorials. See HW4 71. Two common types of problems are below.

i) Write slots, but on top of each slot put a $C(n,r)$ a $P(n,r)$, $n_i!$ or more slots.
Ex: arrange 5 different French books on a top shelf and 6 different Spanish books on a bottom shelf in (5!)(6!) ways. (Slots are "arrange French books on top" and "arrange Spanish books on bottom.")
Ex: obtain a committee with 5 senators and 6 representatives in C(100,5) * C(435,6) ways. (Slots are "pick senators" and "pick representatives.") F04 6, HW3 40a, HW4 38.

ii) Want a license plate with 1 letter followed by 4 digits or 2 letters followed by 4 digits. The word "or" is a key word for "add." Use slots to find the number of 1L 4D plates, use slots to find the number of 2L 4D plates and add to get the result: 26*10*10*10*10 + 26*26*10*10*10*10. S04 5 (TS in the middle or ST in the middle), S04 7, F02 10, S02 15, F01 6, HW4 80

Permutations with Repetition. The number of distinct permutations of n objects, of which n_1 are of the 1st kind, n_2 of the 2nd kind, n_3 of the 3rd kind, ..., n_k of the kth kind is $\dfrac{n!}{n_1!n_2!\cdots n_k!}$ where $n_1 + n_2 + \cdots + n_k = n$.

6) **Common Final Problem.** How many n-letter words (real or imaginary) can be formed from an n-letter word such as MISSISSIPPI? Each letter is a kind: 1 M, 4 I's, 4 S's, 2 P's. There are 11 letters total, so $\dfrac{11!}{1!4!4!2!}$. S04 3, S03 6, Q3 5, HW4 23,24.

7) **Common Final Problem.** Story problem has k groups of indistinguishable objects of size n_i, $i = 1, ..., k$. *Color* is often used, eg green, red and white balls, LIGHTS, or FLAGS. Find the total number of objects $n = n_1 + ... + n_k$. Then the number of ways to arrange the objects is $\dfrac{n!}{n_1!n_2!\cdots n_k!}$. S03 9, HW4 25,26.

8) **Common Final Problem.** A coin is tossed n times. How many outcomes are possible? Solution: 2^n. See E1 9.

9) **Common Final Problem.** A hand of k cards is dealt from a deck of 52 cards. How many k-card hands are there? Solution: $C(52,k)$. See E1 8.