Exam 2 review. Thursday, March 10. Bring a TI 30 calculator but NO NOTES. Emphasis is on sections 1.3, 1.4, 1.5, 1.6, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3; HW2-12; Q1-11. Problems F1) to F13) from Exam 1 review are still fair game.

Types of problems:

From Exam 1 review, 1), 2), 10), 11), 14), 15), 17) and **F1)–F13).**

18) State the definition of the derivative of a function (p. 77, 78, 84). Be able to state the Intermediate value theorem (p. 52)

19) From graph, find points where the derivative does not exist. This could happen if there is a corner, eg $f(x) = |x|$ where the left and right derivatives are not equal, if $f(x)$ is not continuous at $x = c$, or if f has a vertical tangent at $x = c$ (eg $f(x) = x^{1/3}$.) See p. 89.

20) Find the derivatives from the rules, especially the power rule. Know the derivatives of the trig functions and of $f(x) = e^x$. Note that the cofunctions cos, cot, and csc all have minus signs with their derivatives. Know the derivatives of $\ln x$ and $|x|$. Know the derivatives of a^x and $\log_a x$. See p. 98, 111, 115, 117, 164-9. Know the chain rule with these functions.

21) Find the derivative via the product or quotient rule. See p. 107 and 109.

22) Know how to find 2nd, 3rd, etc derivatives, p. 90-1. Know that if $s(t)$ is the position function, then the velocity function $v(t) = s'(t)$ and the acceleration function $a(t) = v'(t) = s"(t)$.

23) Be able to match the graph of a function $f(x)$ with the graph of its derivative $f'(x)$. Practice test 2 page 2, Q8.

24) Section 2.5: The chain rule is **very important** for exam 2 and the final.

25) Know that the derivative of $\ln|g(x)| = g'(x)/g(x)$. Use the fact that $|ab/c| = |a||b|/|c|$ and the properties of logarithms, namely $\ln(ab/c) = \ln(a) + \ln(b) - \ln(c)$ and $\ln(a^p) = p\ \ln(a)$ to write $\ln|g(x)|$ as a sum of logs, then take the derivative of each term. Also $\ln|w_1^{a_1} w_2^{a_2} \cdots w_n^{a_n}| = a_1 \ln|w_1| + a_2 \ln|w_2| + \cdots + a_n \ln|w_n|$. See Q11.

26) Know how to do implicit differentiation, p. 123-6. Be able to find dy/dx when y is a function of x. Then $\frac{d}{dx} g(y) = g'(y)y'$.

27) Know how to find a tangent line to a point (xo,yo) on a curve after you have used implicit differentiation to find dy/dx. In particular, the slope m is found by evaluation the LHS of dy/dx at x = xo and y = yo. Then $yo = mxo + b$.

28) Know how to perform logarithmic differentiation, p. 167-9, especially for problems like $f(x) = x^x$. See back of HW10, Q11.

29) Be able to do related rates problems.

30) Be able to sketch the derivative $f'(x)$ given a graph of $f(x)$. See Q10.

The following new problems are **very important for exam 2** and the final.

F14**) Find the derivative of a sum of power, exponential, trig and logarithmic functions. F13-3a, F14-3b, Q5 6, Q6 3, Q7.

F15***) Find the derivative using the product rule or a combination of the product rule and the chain rule. F13–3b, F14–3a, S15–4c, Q6 4, Q7, S11–3d

F16***) Find the derivative using the quotient rule or a combination of the quotient rule and the chain rule. Remember that the quotient rule is $\dfrac{dn' - nd'}{d^2}$ where d is the denominator and n is the numerator. (F13–3c), F14–3e, S15–4d, Q6 5, Q7, S11–3b.

F17***) Find the derivative using the chain rule. F13–3b(c)de, F14–3ace, S15–4abcd, Q7, Q8, Q9, S11–3acd
The chain rule is $[f(g(x))]' = [f'(g(x))][g'(x)]$.
You should be able to apply it for several layers of functions,
eg [f(g(h(x)))]' = [f'(g(h(x)))][g'(h(x))][h'(x)]. Note that you take the derivative of the outermost function and evaluate it at the inner function. Then you multiply the result by the derivative of the inner function. You need to apply the chain rule to compute this derivative if the inner function is also a composite function.

F18***) Let $y(x)$ be a formula involving x and y. Perform implicit differentiation wrt x to find $y'(x)$. Take the derivative wrt x of both sides using the chain rule. The chain rule is $\frac{d}{dx}g(y) = g'(y)y'$. Generally, $y'(x)$ is a function of both x and y. F13–4, S14–4, F15–6, Q9, Q10, S11–5.

F19) After finding $y'(x)$ as in F18), evaluate $y'(x)$ at a given point (x_o, y_o). Or you could be told to find $y'(x_o)$. This means solve $y(x_o)$ for y_o and then evaluate $y'(x)$ at a given point (x_o, y_o). Q9, Q10?.

F20***) Related rates story problem: draw a picture, find what rate is given and what rate is wanted. Write an equation relating the variables and implicitly differentiate both sides of the equation with respect to **t** (so by the chain rule $\frac{d}{dt}g(w) = g'(w)\frac{dw}{dt}$). Sub in knowns and solve for the desired rate. Rate problems that use the Pythagorean theorem are very common. The less common conical tank problem has 3 variables V, r and h, so use similar triangles to write r as a function of h, then write V as a function of h alone. If the variable (eg distance S) is increasing, then the derivative $\frac{dS}{dt}$ is positive, but if the distance is decreasing, then the derivative $\frac{dS}{dt}$ is negative. Rates are positive, but when $S = 10$ could have distance x decreasing at rate 2 (so $\frac{dx}{dt}|_{S=10} = -2$) or increasing at rate 2 (so $\frac{dx}{dt}|_{S=10} = 2$.) F13–07, F14–6, S15–7, Q10, Q11, S11–2.

F21**) Use logarithmic differentiation to find the derivative of $f(x) = x^{g(x)}$. Assume the domain of $f(x)$ is such that $f(x) > 0$. Note that $\ln(f(x)) = \ln(x^{g(x)}) = g(x)\ln(x)$. Taking derivatives of both sides wrt x gives $\dfrac{f'(x)}{f(x)} = g'(x)\ln(x) + g(x)\dfrac{1}{x}$. So

$f'(x) = f(x)[g'(x)\ln(x) + \dfrac{g(x)}{x}]$.

F13–3f, S15–5, Q11, Q12, S11–3e.