Exam 4 review. Thursday, May 5. Emphasis is on section 2.1 (tangent line); 2.3, 3.3, 3.5, 3.6 (basic derivatives); 2.5 (chain rule); 2.6 (implicit differentiation); 2.7 (related rates); 4.1 (min max); 4.3, 4.4 (increasing, decreasing, concave up down), 4.5 (optimization problems), 5.2, 5.3, 5.4 (definite, indefinite integrals); 5.4 (Fundamental Theorem of Calculus I and II, mean value theorem for integrals); 5.5 (u-substitution), 7.1 (area between two curves), 7.2 (volume by disks and washers), 7.3 (volume by cylindrical shells). From reference pages 5-6 near the back cover of the text (or the table on p. 284) know the derivatives 1-36 (but less emphasis on 22, 23, 24, 34, 35 and 36) and the integrals 1-17 (but less emphasis on 12–15 which you can get with u substitution). No Newton's method (4.6) and no chapter 6. There will be a Riemann sum problem.

From reviews 1, 2 and 3), know how to do all of the problems that are important for the final F1) – F33). Problems (F1, F2, F3, F11, F12, F13, F14, F15, F16, F17, F18, F20, F21, F22, F23, F26, F28, F30, F31, and F32) are especially important and usually have two or three stars.

The following **new problems** are **very important for exam 4** and the final.

F34***) You need to be able to distinguish certain integrals from u-substitution.
a) If the integrand is $(p(x))^2$ where $p(x)$ is a polynomial or sum of powers, then compute the square and use the power rule. F13 11a, S11 6b
b) If the integrand is of the form $p(x)/x^k$ where $p(x)$ is a sum of powers, then divide each power in $p(x)$ by x^k and use the power rule. F15 11a, F14 10a, F13 10a
c) Occasionally a product of trig functions will be a trig function that is easy to integrate. S11 6e
d) (See F23.) If the integrand is $|f(x)|$, find intervals where f is positive and negative. Then $|f(x)| = f(x)$ on the intervals where f is nonnegative and $|f(x)| = -f(x)$ on the intervals where f is negative.

F35***) Indefinite integrals by u-substitution. $I = \int f(g(x))g'(x)dx = F(g(x)) + C$ where $F'(x) = f(x)$. The integrand $f(g(x))g'(x)$ consists of the product of a composite function (a function with an outer function f and an inner function g) with g'(x), the derivative of the inner function. Let $u = g(x)$. Then $du = g'(x)dx$ and $I = F(u) + C = F(g(x)) + C$. Note that F is the antiderivative of the outer function f.
S15 11bcd, F14 10bc, F13 10bcde, S11 6acd

F36***) Find the definite integral by u-substitution in either of two ways:
a) $I = \int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du = F(u)|_{g(a)}^{g(b)} = F(g(x))|_a^b = F(g(b)) - F(g(a))$, or
b) $I = \int_a^b f(g(x))g'(x)dx = \int_c^d f(u)du = F(u)|_c^d = F(d) - F(c)$.
For a) and b), $F'(x) = f(x)$, $u = g(x)$, $du = g'(x)dx$, $d = g(b)$, and $c = g(a)$. S15 12a, F14 11ab, F13 11b, S11 7abc

Note: for both F35) and F36), if the integrand is $f(g(x))kg'(x)$ where k is a constant, then pull k outside the integral, possibly by multiplying the integral by one in the form of $1 = k/k$. In other words, you can use substitution if the derivative $g'(x)$ is present up to a constant. Also $\int(f_1(ax) + f_2(bx))dx = \int f_1(ax)dx + \int f_2(bx)dx$. Use $u = ax$ and $u = bx$ for the 1st and 2nd integrals. S15 11c, F14 10c, F13 10b.

F37**) For u-substitution, the integrand has to have the form $[f(g(x))]h(x)$. Then $u = g(x)$. To make $h(x)dx = g'(x)dx$, sometimes the integral has to be multiplied by one in the form c/c where c is some constant. In particular,
a) If $g(x) = ax + b$, then $h(x) \equiv 1$ (or any other constant).
b) If the integrand is $h(x)/g(x)$, try $u = g(x)$.
c) If the integrand is $h(x)/(g(x))^k$, try $u = g(x)$.
d) If the integrand is $h(x)e^{g(x)}$, try $u = g(x)$.

F38***) Suppose the integrand has the form a) $x(a + x)^r$ then $u = a + x$, du = dx and $x = u - a$.
b) $x(x - a)^r$ then $u = x - a$, du = dx and $x = u + a$.
c) $x/(a + x)^r$ then $u = a + x$, du = dx and $x = u - a$.
d) $x/(x - a)^r$ then $u = x - a$, du = dx and $x = u + a$.
e) change a plus to a minus and vice verca in a)-d).
Notice that

$$\int x(x-a)^r dx = \int (u+a)u^r du = \int u^{r+1} + au^r du = \frac{(x-a)^{r+2}}{r+2} + a\frac{(x-a)^{r+1}}{r+1} + C$$

for $r \neq -2, -1$ and

$$\int \frac{xdx}{(x-a)^r} = \int \frac{(u+a)}{u^r} du = \int u^{-r+1} + au^{-r} du = \frac{(x-a)^{-r+2}}{-r+2} + a\frac{(x-a)^{-r+1}}{-r+1} + C$$

for $r \neq 1, 2$. Often $(x - a)^r$ will be a radical such as $\sqrt{x+1}$ or $\sqrt[3]{x-1}$. Problems like find $\int x(1 \pm x)^r dx$ are especially common. S15 12a, F14 11a, F13 10e, S11 6a.

F39) Often u-substitution can also be used if the integrand has to have the form $[h(f(x))][k(g(x))]$. If $k(g(x))$ is more complicated than $h(f(x))$, then try $u = g(x)$.

An important special case is $k(t) = h(t) = 1/t$, that is, the integrand has the form $[1/(f(x))][1/g(x)]$. If $g(x)$ is more complicated than $f(x)$, try $u = g(x)$.

F40***) To find the area between two curves $f(x)$ and $g(x)$ on [a,b], find $k \geq 1$ intervals $[x_{i-1}, x_i]$ where one of the curves f or g is the top curve. Here $x_0 = a$ and $x_k = b$. Then the area between curves f and g is

$A = \int_a^b |f(x) - g(x)|dx = \sum_{i=1}^k \int_{x_{i-1}}^{x_i}$ (top curve - bottom curve) dx.

Usually one curve, say f, satisfies $f(x) \geq g(x)$ on [a,b]. Then

$$A = \int_a^b |f(x) - g(x)|dx = \int_a^b (f(x) - g(x))dx.$$

EACH INTERVAL IS A DEFINITE INTEGRAL THAT GIVES A POSITIVE NUMBER. S15 13a, F14 12a, F13 12a, S11 9

F41***) Volume by disks and washers. The volume of a solid of revolution is obtained by rotating y = f(x) = R(x) on [a,b] about the x-axis is $V = \pi \int_a^b [R(x)]^2 dx$. If the curve $x = g(y) = R(y)$ on [c,d] is rotated about the y-axis, then $V = \pi \int_c^d [R(y)]^2 dy$. If the plane region formed by the outer radius R(x) and the inner radius r(x) is rotated about the x-axis, then the volume by washers is given by $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$. If the plane region formed by the outer radius R(y) and the inner radius r(y) is rotated about the y-axis, then the volume is $V = \pi \int_c^d ([R(y)]^2 - [r(y)]^2) dy$. Determine the axis of revolution and draw a line segment **perpendicular** to the axis of revolution through the plane curve. The inner radius r is the smaller distance (where the line segment 1st hits the 2 curves) and the outer radius R is the larger radius. (If the axis is one curve and the other curve is the function rotated about the axis, then the radius = top curve − bottom curve for an axis parallel to the x-axis. The radius = right curve − left curve if the axis is parallel to the y-axis. Two curves are revolved about the axis, and R corresponds to the curve farthest from the axis.)

Occasionally you need to solve y = f(x) for x = R(y) and sometimes the radius R is defined differently over different intervals. Note that the formula for disks is obtained from the formula for washers with $r(x) \equiv 0$. Use dx if the axis of revolution is parallel to the x-axis and use dy if the axis of revolution is parallel to the y-axis. Usually you just set up the integral, but sometimes you do need to evaluate the volume. See S15 13b, S11 13. Can do integral by F41) or F42), but usually one method is easier.

F42***) Volume by shells. Assume that the region is in the 1st quadrant. If the region is rotated about a horizontal or x-axis of revolution then $V = 2\pi \int_c^d r(y)h(y)dy$. If the region is rotated about a vertical or y-axis of revolution, then $V = 2\pi \int_a^b r(x)h(x)dx$. Draw a line segment entirely within the region that is **parallel** to the axis of revolution. The r is the distance from the axis of revolution to the line segment (r is also the radius of the cylindrical shell) while h is the height of the line segment. Often h is the difference between a top and a bottom curve or a right and a left curve. For the x-axis, $r(y) = y$ and for the y-axis $r(x) = x$. Use dy if the axis of revolution is parallel to the x-axis and use dx if the axis of revolution is parallel to the y-axis. Usually you simply set up the integral. See S15 13c, F14 12b, S13 bc, S11 14.

F43**) The integral of an odd function over $[-a, a]$ is equal to 0. Note that $\tan(x)$ and $\sin(x)$ are odd functions: $f(-x) = -f(x)$ while $\cos(x)$ is an even function: $f(-x) = f(x)$. Also x^k is odd if k is a positive odd integer while x^k is even if k is a positive even integer. An odd function divided by an even function is an odd function. S15 12b, F14 11b.