Final is Wed. May 7, 12:50-2:50. **You are allowed 15 sheets of notes and a calculator.** The final is cumulative, so you should know everything on the first 4 reviews. This is material not on those reviews.

184) Suppose S_t is the price of the underlying asset for $0 \leq t \leq T$. Then the theoretical price of the long forward at time t is $P_L(t) = S_t - S_0 e^{rt} = S_t - S_0(1+i)^t$.

The theoretical price of the short forward at time t is
$P_S(t) = S_0 e^{rt} - S_t = S_0(1+i)^t - S_t$.

Note that the two prices sum to 0. The theoretical prices are fairly accurate estimates of the market prices at time t.

185) There are 4 common ways to buy stock. If prices below are for one share of stock and J shares of stock are purchased, then multiply the payments by J. Recall that r is the risk free interest rate compounded continuously = risk free force of interest. A share of stock has (continuous) dividend yield (rate) δ if the number of shares grows continuously: so 1 share will grow to $e^{\delta T}$ shares at time T. Continuous dividend yield is common for shares of index funds or mutual funds. You may need to substitute $1 + i = e^r$, et cetera, where i is the risk free interest rate. The payment price for the stock assumes that the dividend payment times and amounts are known. (More advanced pricing techniques have dividend uncertainty reflected in the calculated stock price.)

a) **Outright purchase**: receive stock at time $t = 0$ and pay S_0 at time $t = 0$.

b) **Forward contract**: receive stock at time T and pay at time T (buyer does not receive dividends paid in $[0, T)$):

i) pay $F_{0,T} = S_0 e^{rT} = S_0(1+i)^T$ for nondividend paying stock,

ii) pay $F_{0,T} = S_0 e^{rT} -$ FV(dividends) $= S_0(1+i)^T -$ FV(dividends) for stock which pays discrete dividends (eg quarterly),

iii) pay $F_{0,T} = S_0 e^{(r-\delta)T}$ when stock has dividend yield δ.

c) **Prepaid forward = prepay**: receive stock at time T and pay at time 0 (buyer does not receive any dividends paid in $[0, T)$):

i) pay $F^P_{0,T} = S_0$ for nondividend paying stock,

ii) pay $F^P_{0,T} = S_0 -$ PV(dividends) for stock which pays discrete dividends,

iii) pay $F^P_{0,T} = S_0 e^{-\delta T}$ when stock has dividend yield δ.

d) **Fully leveraged position**: Receive stock at time 0, pay $S_0 e^{rT} = S_0(1+i)^T$ at time T. (Borrow S_0 to pay for the stock at time 0, then repay the loan at time T.)

The forward price = (prepaid forward price) $(1+i)^T$ = (prepaid forward price) e^{rT}.

186) For the buyer, the forward payoff is $S_T - F_{0,T}$ and the prepaid forward payoff is $S_T - F^P_{0,T}$. The broker's (trader's or amrket maker's) payoffs are $F_{0,T} - S_T$ or $F^P_{0,T} - S_T$. If the payoffs (= profits) are for 1 share of stock and the contract is for J shares, multiply the payoffs by J.

187) An **in the money option** is an option, if exercised now, would have a positive payoff. An **at the money option** is an option, if exercised now, would have approximately 0 payoff. An **out of the money option** is an option, if exercised now, would have a negative payoff.

By "immediately," assume the current time is t where $0 \leq t \leq T$. In determining "moneyness" act as if a European option could be executed immediately (or sold for the strike price at time 0).

A purchased call is in the money if the current spot price is greater than the strike price: $S_t > K$. The long call is out of the money if $S_t < K$

A purchased put is in the money if the current spot price is less than the strike price: $S_t < K$. The long put is out of the money is $S_t > K$.

188) An *asset price contingency* is the condition for the underlying asset to be bought or sold. The asset is exercised if it is bought or sold for an option. The asset is always bought or sold for a forward.

189) The following table gives the maximum "loss" (negative profit) and maximum gain for some derivatives. The entries that are not "unlimited" occur if $S_T = 0$ or if the payoff $= 0$.

derivative	maximum "loss"	maximum gain
long forward	$-F_{0,T}$	unlimited
short forward	unlimited	$F_{0,T}$
long call	$-FV(C)$	unlimited
short call	unlimited	$FV(C)$
long put	$-FV(P)$	$K - FV(P)$
short put	$FV(P) - K$	$FV(P)$

190) A **futures contract** is similar to a forward but has several differences. Futures are widely bought and sold daily in markets during the life $[0, T]$ of the contract. Forwards are typically sold by brokers. The amount needed to open a futures account is called an *initial margin*. To maintain order in futures markets, there are daily limits on the movement of future prices. At the end of each trading day, a futures account is *marked-to-market* which means that any profit or loss resulting from a change in the future's price from the previous trading day's close is added or deducted from the account balance. The holder of the *long futures contract* is obligated to buy and the holder of the *short futures contract* is obligated to sell the asset at expiry time T.

day	futures price $S_t = S_{t-1} + C_t$	price change $C_t = S_t - S_{t-1}$	credit or debit JC_t	margin account balance $B_t = B_{t-1}e^{r/365} + JC_t$
0	S_0			B_0
1	S_1	C_1	JC_1	B_1
2	S_2	C_2	JC_2	B_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T-1$	S_{T-1}	C_{T-1}	JC_{T-1}	B_{T-1}
T	S_T	C_T	JC_T	B_T

191) The above table is used to compute the balance $B_t = B_{t-1}e^{r/365} + JC_t$ in the margin account where 1 day = (1/365)th of a year. Replace 365 by 52 if weeks are used and by 12 if months are used. Let S_t be the price of 1 unit of the future on day t (at the close of the market). Let $C_t = S_t - S_{t-1}$ be the price change of 1 unit of the future. Let J be the multiplier = number of units in the future contract so the size of the contract is JS_0. Then $B_0 = D\%$ (size) $= D\% JS_0$, eg $D\% = 10\%$. Note that $S_T = S_0 + C_1 + C_2 + \cdots + C_T$. Then long future profit $= B_T - B_0 e^{rT/365} = B_T - AV(B_0)$.

192) If a long forward has $F_{0,T} = S_0$, then the profit on the long forward is $J(S_T - S_0)$ which is the profit of the long future when there is no interest: $\delta = 0$. (With interest, $F_{0,T} = S_0 e^{rT}$, assuming no income, such as dividends, from the asset during $[0, T]$.)

193) The purchaser of a futures contract may be required to add funds to the margin account if B_t falls below a predefined level called the maintenance margin.

194) A **swap** is a contract that covers a stream of payments over a period of time. (Options and forwards are single payment swaps.) A swap is an agreement to exchange (swap) one set of payments for another set of payments over time.

195) Let $P(0, t_i) = (1 + s_i)^{-t_i} = v_{s_i}^{t_i}$ be the PV of a \$1 zero coupon bond (treasury bill) maturing at time t_i with (risk free) spot rate s_i.

196) For a commodity swap, suppose a commodity is needed at times $t_1, ..., t_n$ and a swap contract is available with guaranteed prices $F_{t_1}, ..., F_{t_n}$ (eg from n forward contracts with expiration dates $T_i = t_i$). A *prepaid swap* has price = PV of the guaranteed prices at the risk free interest spot rates s_{t_i}. Thus *prepaid price* $= \sum_{i=1}^n F_{t_i} v_{s_i}^{t_i}$.

197) The customary way to handle payments of a swap is to make level payments R at times t_i such that the PV of the payments = prepaid price. Thus $\sum_{i=1}^n R v_{s_i}^{t_i} = \sum_{i=1}^n F_{t_i} v_{s_i}^{t_i}$ where the level payment is the **swap price** $R = \overline{F} = \dfrac{\sum_{i=1}^n F_{t_i} v_{s_i}^{t_i}}{\sum_{i=1}^n v_{s_i}^{t_i}} = \dfrac{\sum_{i=1}^n P(0, t_i) F_{t_i}}{\sum_{i=1}^n P(0, t_i)}$. Be able to find R given the F_{t_i} and a table of the t_i and s_{t_i}. Often $t_j = j$.

198) Let $r_0(t_{i-1}, t_i)$ be the forward rate that is effective from time t_{i-1} to time t_i. Then $r_0(j, j+1) = i(j, j+1) = f_j$, the j year forward rate for year $j+1$ starting at time j. See 133) - 135). Hence $r_0(j, j+1) = f_j = \dfrac{(1 + s_{j+1})^{j+1}}{(1 + s_j)^j} - 1$. Suppose nonlevel interest payments $r_0(t_{i-1}, t_i)$ are made at times t_i for $i = 1, ..., n$ where $t_0 = 0$. If level interest payments R are made at times t_i such that the level payment stream and nonlevel payment stream have the same PV, then $\sum_{i=1}^n R v_{s_i}^{t_i} = \sum_{i=1}^n r_0(t_{i-1}, t_i) v_{s_i}^{t_i}$ where *swap rate*
$$R = \frac{\sum_{i=1}^n r_0(t_{i-1}, t_i) v_{s_i}^{t_i}}{\sum_{i=1}^n v_{s_i}^{t_i}} = \frac{\sum_{i=1}^n P(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=1}^n P(0, t_i)} = \frac{1 - P(0, t_n)}{\sum_{i=1}^n P(0, t_i)}.$$

199) To create a **synthetic long forward**, at time 0 buy a purchased (long) call and sell a written (short) put for the underlying asset where both the call and the put expire at time T with strike price K. The synthetic long forward price $= C - P = C(K, T) - P(K, T)$ where C is the call premium and P is the put premium. Then the synthetic long forward payoff $= S_T - K =$ long forward payoff. The synthetic long forward profit $= S_T - K - FV(C - P) = S_T - K - (C - P)(1 + i)^T = S_T - K - (C - P)e^{rT}$. The purchaser of the synthetic long forward buys the asset at time T for the strike price K. Here i is the effective annual interest rate and r is the annual interest rate compounded continuously = force of interest. As usual, other formulas may be needed to compute $FV(C - P)$. The profit and payoff formulas assume that the asset earns no income (eg dividends) during $[0, T]$.

200) To create a **synthetic short forward**, at time 0 buy a purchased (long) put and sell a written (short) call for the underlying asset where both the call and the put expire at time T with strike price K. The synthetic short forward price $= P - C = P(K, T) - C(K, T)$. Then the synthetic short forward payoff $= K - S_T =$ short forward payoff. The synthetic short forward profit $= K - S_T - FV(P - C) =$
$K - S_T - (P - C)(1 + i)^T = K - S_T - (P - C)e^{rT}$. The purchaser of a synthetic short forward sells the asset at time T for the strike price K.

201) The **put call parity:** $C - P = PV(F_{0,T} - K)$. The put call parity relationship

arises since the PV of the cash outflows to buy an asset should be the same, regardless of the method. A long forward with cash outflow of $F_{0,T}$ at time T has a PV of $PV(F_{0,T}) = F_{0,T}(1+i)^{-T} = F_{0,T}v_i^T$. A long synthetic forward has a cash outflow of $C - P$ at time 0 and a cash outflow of K at time T. The PV of these outflows is $C - P + PV(K)$. Using $S_0 = C - P + PV(K) = PV(F_{0,T})$ can give several relationships, including the put call parity and $C - P = S_0 - PV(K)$, $S_0 = PV(F_{0,T})$, and $F_{0,T} = FV(S_0) = S_0 e^{rT} = S_0(1+i)^T$.

202) The **put call parity:** $C - P = PV(F_{0,T} - K)$ holds with

i) $F_{0,T} = S_0 e^{rT} = S_0(1+i)^T$ for nondividend paying stock,

ii) $F_{0,T} = S_0 e^{rT} -$ FV(dividends) $= S_0(1+i)^T -$ FV(dividends) for stock which pays discrete dividends (eg quarterly),

iii) $F_{0,T} = S_0 e^{(r-\delta)T}$ when stock has dividend yield δ.

203) The opportunity to make a sure profit with no risk is called **arbitrage**. Such opportunities can't survive long in the marketplace. A fundamental assumption made in pricing derivatives is that arbitrage is not possible. Hence this pricing is called **no-arbitrage pricing**. This pricing is used to derive the put call parity.

204) An **off-market (long) forward** has a premium $C - P$ and forward price (for the asset) of $F_{0,T} = K$, the same cash flows as a synthetic long forward.

205) Let L = long (purchased) and S = short (written) so LCall(K) is a long Call with stike price K and and SPut(K) is a short put with strike price K. Memorize the payoff graphs for the LCall, SCall, LPut and SPut. For combinations of options, i) want to know the strategy, ii) know the shape of the profit graph, and iii) know the combination of options that make up the derivative.

The (short) written profit graph will be the mirror image (about the x-axis) of the (long) purchased profit graph, and the profit is the combination of profits that make up the derivative. Written derivative profit = − purchased derivative profit.

206) A (long) **straddle** i) bets price will either go up or down from K, ii) is V shaped, iii) = LPut(K) + LCall(K).
So straddle profit $= \max(0, K - S_T) - FV(P) + \max(0, S_T - K) - FV(C)$.

A **written straddle** i) bets price will be near K, ii) shape is an upside down V, iii) = SPut(K) + SCall(K).

207) A strangle i) bets price will either go up or down from K, ii) decreases linearly, then flat from $K-D$ to $K+D$, then increases linearly, iii) = LPut($K-D$) + LCall($K+D$). So strangle profit $= \max(0, K - D - S_T) - FV(P(K-D)) + \max(0, S_T - K - D) - FV(C(K+D))$ where $P(K-D)$ is the premium of a put with strike price $K - D$ and $C(K+D)$ is the premium of a call with strike price $K + D$.

208) Bull spread = LCall(K_1)+ SCall(K_2) , $K_2 > K_1$

Bear spread = SCall(K_1)+ LCall(K_2) , $K_2 > K_1$

Collar = LPut(K_1)+ SCall(K_2), $K_2 > K_1$

```
      K1    K2    K3                Kn
 -------------------------------------------------
 |     |     |     |       ...       |     |
 t0    t1    t2    t3                tn    t_(n+1)=T
```

209) Consider the above time diagram where deposits of K_i are made at times $t_1, t_2, ..., t_n$. Often $t_j = j$ for $j = 0, 1, ..., n, n+1$ and often $K_j = K$ for $j = 1, ..., n$. Sometimes $t_0 = t_1$ and $T = t_{n+1} = t_n$. The table below gives $PV(t_0)$ and $AV(T)$ for some important methods.

	$PV(t_0)$	$AV(T)$
compound	$\sum_{j=1}^{n} K_j(1+i)^{-(t_j-t_0)}$	$\sum_{j=1}^{n} K_j(1+i)^{(T-t_j)}$
simple	$\sum_{j=1}^{n} K_j[1+i(t_j-t_0)]^{-1}$	$\sum_{j=1}^{n} K_j[1+i(T-t_j)]$
$a(t)$ not simple	$\sum_{j=1}^{n} K_j \frac{a(t_0)}{a(t_j)}$	$\sum_{j=1}^{n} K_j \frac{a(T)}{a(t_j)}$
spot interest	$\sum_{j=1}^{n} K_j(1+s_j)^{-(t_j-t_0)}$	$\sum_{j=1}^{n} K_j(1+s_j)^{(T-t_j)}$
immediate, $K_j = K, t_j = j+d$, $T = t_n$	$K\ a_{\overline{n}\vert}$	$K\ s_{\overline{n}\vert}$
due $K_j = K, t_j = j+d$, $t_0 = t_1$	$K\ \ddot{a}_{\overline{n}\vert}$	$K\ \ddot{S}_{\overline{n}\vert}$