Math 402 Exam 3 is Wed. April 27. **You are allowed 12 sheets of notes and a calculator.** The exam emphasizes HW8-10 and Q8-10.

Chapter 7

A policy value or NLP terminal reserve $_tV = E(\,_tL) = E(Z_{x+t} - PY_{x+t}) = A_{x+t} - Pa_{x+t}$ for insurance. So the terminal reserve = (APV of insurance or annuity at time $x+t$) − (APV of premiums yet to be paid at time $x+t$) = value of policy at time t. $_0L = L$ from ch. 6. **If the insurance benefit is B instead of 1, multiply the unit benefit formula $_tV$ by B.** Formulas for $_tV$ are for unit benefit except 151) and 152).

139) Given $X > x+t$, $T_{x+t} = T_x - t$ for $t > 0$ while $K_{x+t} = K_x - t$ for integer t.

140) Continuous funding, continuous payment whole life insurance: Given $T_x > t$, $_t\overline{L}(\overline{A}_x) = v^{T_x - t} - [\overline{P}(\overline{A}_x)]\ \overline{a}_{\overline{T_x - t}|} = \overline{Z}_{x+t} - [\overline{P}(\overline{A}_x)]\ \overline{Y}_{x+t}$. The NLP terminal reserve $_t\overline{V}(\overline{A}_x) = E[\,_t\overline{L}(\overline{A}_x)] = \overline{A}_{x+t} - [\overline{P}(\overline{A}_x)]\ \overline{a}_{x+t} = 1 - \dfrac{\overline{a}_{x+t}}{\overline{a}_x} = [\overline{P}(\overline{A}_x)][\overline{s}_{x:\overline{t}|}] - \,_t\overline{k}_x = 1 - [\overline{P}(\overline{A}_x) + \delta]\ \overline{a}_{x+t} = [\overline{P}(\overline{A}_{x+t}) - \overline{P}(\overline{A}_x)]\ \overline{a}_{x+t}$.

$\mathrm{Var}[\,_t\overline{\mathrm{L}}(\overline{\mathrm{A}}_\mathrm{x})] = \mathrm{V}[\,_t\overline{\mathrm{L}}(\overline{\mathrm{A}}_\mathrm{x})] = \left(\dfrac{1}{\delta\ \overline{\mathrm{a}}_\mathrm{x}}\right)^2\ [\,^2\overline{\mathrm{A}}_{\mathrm{x+t}} - (\overline{\mathrm{A}}_{\mathrm{x+t}})^2] = \dfrac{^2\overline{\mathrm{A}}_{\mathrm{x+t}} - (\overline{\mathrm{A}}_{\mathrm{x+t}})^2}{(1 - \overline{\mathrm{A}}_{\mathrm{x+t}})^2}$.

141) Discrete whole life insurance with annual premiums. Given $K_x \geq t$, i) whole life: $_tL_x = v^{K_x+1-t} - P_x\ \ddot{a}_{\overline{K_x+1-t}|} = Z_{x+t} - P_x\ \ddot{Y}_{x+t}$. Then $_0L_x = L_x$. The NLP terminal reserve $_tV_x = E(\,_tL_x) = A_{x+t} - P_x\ \ddot{a}_{x+t} = 1 - \dfrac{\ddot{a}_{x+t}}{\ddot{a}_x} = \dfrac{P_{x+t} - P_x}{P_{x+t} + d} = [1 - \dfrac{P_x}{P_{x+t}}]\ A_{x+t} = 1 - (P_x + d)\ddot{a}_{x+t}$.

$\mathrm{Var}(\,_t\mathrm{L}_\mathrm{x}) = \mathrm{V}(\,_t\mathrm{L}_\mathrm{x}) = \left(1 + \dfrac{\mathrm{P}_\mathrm{x}}{\mathrm{d}}\right)^2\ [\,^2\mathrm{A}_{\mathrm{x+t}} - (\mathrm{A}_{\mathrm{x+t}})^2] = \left(\dfrac{1}{\mathrm{d}\ \ddot{\mathrm{a}}_\mathrm{x}}\right)^2\ [\,^2\mathrm{A}_{\mathrm{x+t}} - (\mathrm{A}_{\mathrm{x+t}})^2]$
$= \mathrm{Var}[\,_t\mathrm{L}_\mathrm{x}] = \dfrac{^2\mathrm{A}_{\mathrm{x+t}} - (\mathrm{A}_{\mathrm{x+t}})^2}{(1 - \mathrm{A}_\mathrm{x})^2}$.

ii) n year term: $_tV^1_{x:\overline{n}|} = A^1_{x+t:\overline{n-t}|} - P^1_{x:\overline{n}|}\ \ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$.

iii) n year pure endowment: $_tV_{x:\overline{n}|}^{\ \ \ 1} = A_{x+t:\overline{n-t}|}^{\ \ \ \ \ 1} - P_{x:\overline{n}|}^{\ \ \ 1}\ \ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$.

iv) n year endowment: $_tV_{x:\overline{n}|} = A_{x+t:\overline{n-t}|} - P_{x:\overline{n}|}\ \ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$.

v) h-pay whole life insurance (h premiums): $^h_tV_x = A_{x+t} - \,_hP_x\ \ddot{a}_{x+t:\overline{h-t}|}$ for $t < h$, and $^h_tV_x = A_{x+t}$ for $t \geq h$.

vi) n-year deferred insurance with n premiums: $^n_tV(\,_n|A_x) = \,_{n-t}|A_{x+t} - \,_nP(\,_n|A_x)\ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$, and $^n_tV(\,_n|A_x) = A_{x+t}$ for $t \geq n$.

vii) n-year deferred annuity with n premiums: $_tV(\,_n|\ddot{a}_x) = \,_{n-t}|\ddot{a}_{x+t} - P(\,_n|\ddot{a}_x)\ddot{a}_{x+t:\overline{n-t}|}$.

These formulas are for integer t.

142) Fully continuous insurance (or annuity) has continuous payment and continuous funding with a continuous insurance (or annuity). Formulas are valid for real $t > 0$ and the formulas tend to be the same as those in 141) after barring V, A, and a. Assume $T_x > t$.

i) whole life: see 140). $_t\overline{V}(\overline{A}_x) = 1 - \dfrac{\overline{a}_{x+t}}{\overline{a}_x}$ and $\mathrm{Var}[\,_t\overline{\mathrm{L}}(\overline{\mathrm{A}}_\mathrm{x})] = \dfrac{^2\overline{\mathrm{A}}_{\mathrm{x+t}} - (\overline{\mathrm{A}}_{\mathrm{x+t}})^2}{(1 - \overline{\mathrm{A}}_{\mathrm{x+t}})^2}$.

ii) n year term: $_t\overline{V}(\overline{A}^1_{x:\overline{n}|}) = \overline{A}^1_{x+t:\overline{n-t}|} - \overline{P}(\overline{A}^1_{x:\overline{n}|})\ \overline{a}_{x+t:\overline{n-t}|}$ for $t < n$.

iii) n year pure endowment: ${}_t\overline{V}_{\substack{1\\x:\overline{n}|}} = A_{\substack{1\\x+t:\overline{n-t}|}} - \overline{P}_{\substack{1\\x:\overline{n}|}}\ \overline{a}_{x+t:\overline{n-t}|}$ for $t < n$. Note that there is no bar over the A.

iv) n year endowment: ${}_t\overline{V}(\overline{A}_{x:\overline{n}|}) = \overline{A}_{x+t:\overline{n-t}|} - \overline{P}(\overline{A}_{x:\overline{n}|})\ \overline{a}_{x+t:\overline{n-t}|}$ for $t < n$.

v) h-pay whole life insurance: ${}^h_t\overline{V}(\overline{A}_x) = \overline{A}_{x+t} - {}_h\overline{P}(\overline{A}_x)\ \overline{a}_{x+t:\overline{h-t}|}$ for $t < h$, and ${}^h_t\overline{V}(\overline{A}_x) = \overline{A}_{x+t}$ for $t \geq h$.

vi) n-year deferred annuity: ${}_t\overline{V}(\ {}_n|\overline{a}_x) = {}_{n-t}|\overline{a}_{x+t} - \overline{P}(\ {}_n|\overline{a}_x)\overline{a}_{x+t:\overline{n-t}|}$.

143) Note that fully continuous insurance and annuities tend to have the insurance or annuity in parentheses for the reserve ${}_tV$ and the premium P. For discrete insurance and annuities, the parentheses are dropped but the subscripts are used for the reserve ${}_tV$ and the premium P. An exception is discrete deferred insurance and annuities which do use parentheses.

144) **Know:** Suppose the equivalence principle is used to determine premiums.

i) fully continuous whole life: $\mathrm{Var}[\ {}_t\overline{\mathrm{L}}(\overline{\mathrm{A}}_\mathrm{x})] = \dfrac{{}^2\overline{\mathrm{A}}_{\mathrm{x+t}} - (\overline{\mathrm{A}}_{\mathrm{x+t}})^2}{(1 - \overline{\mathrm{A}}_\mathrm{x})^2}$. See 140).

ii) fully continuous n-year endowment insurance $t < n$:
$\mathrm{Var}[\ {}_t\overline{\mathrm{L}}(\overline{\mathrm{A}}_{\mathrm{x:\overline{n}|}})] = \dfrac{{}^2\overline{\mathrm{A}}_{\mathrm{x+t:\overline{n-t}|}} - (\overline{\mathrm{A}}_{\mathrm{x+t:\overline{n-t}|}})^2}{(1 - \overline{\mathrm{A}}_{\mathrm{x:\overline{n}|}})^2}$.

iii) discrete whole life: $\mathrm{Var}[\ {}_t\mathrm{L}_\mathrm{x}] = \dfrac{{}^2\mathrm{A}_{\mathrm{x+t}} - (\mathrm{A}_{\mathrm{x+t}})^2}{(1 - \mathrm{A}_\mathrm{x})^2}$. See 138) i).

iv) discrete n-year endowment insurance with integral $t < n$:
$\mathrm{Var}[\ {}_t\mathrm{L}_{\mathrm{x:\overline{n}|}}] = \dfrac{{}^2\mathrm{A}_{\mathrm{x+t:\overline{n-t}|}} - (\mathrm{A}_{\mathrm{x+t:\overline{n-t}|}})^2}{(1 - \mathrm{A}_{\mathrm{x:\overline{n}|}})^2}$.

Take $t = 0$ to get $\mathrm{Var}[\ {}_0\mathrm{L}] = \mathrm{Var}[\mathrm{L}]$ for chapter 6.

145) Continuous payment, continuous funding, discrete insurance puts bars over V, P, and a. So ${}_t\overline{V}_x = A_{x+t} - \overline{P}_x\overline{a}_{x+t}$.

146) Policy values = NLP terminal reserves for continuous payment insurance with annual premiums put a bar over A.

i) whole life: ${}_tV(\overline{A}_x) = \overline{A}_{x+t} - P(\overline{A}_x)\ \ddot{a}_{x+t}$.

ii) n year term: ${}_tV(\overline{A}^1_{x:\overline{n}|}) = \overline{A}^1_{x+t:\overline{n-t}|} - P(\overline{A}^1_{x:\overline{n}|})\ \ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$.

iii) n year endowment: ${}_tV(\overline{A}_{x:\overline{n}|}) = \overline{A}_{x+t:\overline{n-t}|} - P(\overline{A}_{x:\overline{n}|})\ \ddot{a}_{x+t:\overline{n-t}|}$ for $t < n$.

iv) h-pay n-year term (h premiums): ${}^h_tV(\overline{A}^1_{x:\overline{n}|}) = \overline{A}^1_{x+t:\overline{n-t}|} - {}_hP(\overline{A}^1_{x:\overline{n}|})\ \ddot{a}_{x+t:\overline{h-t}|}$ for $t < h < n$, and ${}^h_tV(\overline{A}^1_{x:\overline{n}|}) = \overline{A}^1_{x+t:\overline{n-t}|}$ for $h < t < n$.

147) At time t, let ${}_tV$ be the policy value = NLP terminal reserve, A_{x+t} be the APV of the insurance, a_{x+t} be the APV of the remaining unit premiums, and P_{x+t} be the premium for $(x + t)$.

i) The prospective formula is ${}_tV = A_{x+t} - P_x a_{x+t}$.

ii) The premium difference formula is ${}_tV = a_{x+t}(P_{x+t} - P_x)$.

iii) The paid up insurance formula is ${}_tV = A_{x+t}\left(1 - \dfrac{P_x}{P_{x+t}}\right)$.

148) For example, i) discrete whole life: ${}_tV_x = A_{x+t} - P_x\ \ddot{a}_{x+t} = \ddot{a}_{x+t}(P_{x+t} - P_x) = A_{x+t}\ [1 - \frac{P_x}{P_{x+t}}] = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$.

ii) fully continuous whole life: ${}_t\overline{V}(\overline{A}_x) = \overline{A}_{x+t} - [\overline{P}(\overline{A}_x)]\ \overline{a}_{x+t} =$
$\overline{a}_{x+t}[\overline{P}(\overline{A}_{x+t}) - \overline{P}(\overline{A}_x)] = \overline{A}_{x+t}\ [1 - \dfrac{\overline{P}(\overline{A}_x)}{\overline{P}(\overline{A}_{x+t})}] = 1 - \dfrac{\overline{a}_{x+t}}{\overline{a}_x}$.

149) Using the same notation as in 147), for whole life and endowment insurance,

i) the annuity ratio formula is ${}_tV = \left(1 - \dfrac{a_{x+t}}{a_x}\right)$.

ii) The insurance ratio formula is ${}_tV = \dfrac{A_{x+t} - A_x}{1 - A_x}$.

iii) The premium ratio formula is ${}_tV = \dfrac{P_{x+t} - P_x}{P_{x+t} + d}$, but replace d by δ for fully continuous insurance.

Know how to use i) and ii) to calculate a reserve for discrete whole life insurance when mortality follows the illustrative life table where ${}_tV_x = \left(1 - \dfrac{\ddot{a}_{x+t}}{\ddot{a}_x}\right) = \dfrac{A_{x+t} - A_x}{1 - A_x}$.

For fully continuous whole life insurance, ${}_t\overline{V}(\overline{A}_x) = 1 - \dfrac{\overline{a}_{x+t}}{\overline{a}_x} = \dfrac{\overline{A}_{x+t} - \overline{A}_x}{1 - \overline{A}_x} = \dfrac{\overline{P}(\overline{A}_{x+t}) - \overline{P}(\overline{A}_x)}{\overline{P}(\overline{A}_{x+t}) + \delta}$.

150) If $T_x \sim EXP(\mu)$, then ${}_t\overline{V}(\overline{A}_x) = 0$ since $\overline{a}_x = \overline{a}_{x+t} = \dfrac{1}{\mu + \delta}$ and $\overline{A}_x = \overline{A}_{x+t} = \dfrac{\mu}{\mu + \delta}$ do not depend on t. See 28). Also, $Var({}_t\overline{L}(\overline{A}_x) = {}^2\overline{A}_x = \dfrac{\mu}{\mu + 2\delta}$.

151) A recursive formula is ${}_{t+1}V = ({}_tV + P)(1 + i) - q_{x+t}(B - {}_{t+1}V)$ where B is the benefit paid, often $B = 1$ unit.

152) Let π_t be the premium paid at time $t = 0, 1, ...$ and let b_k be the benefit paid at time k if the death occurs in the kth year of the policy for $k = 1, 2, ...$ Then
$({}_{k-1}V + \pi_{k-1})(1 + i) = q_{x+k-1}(b_k - {}_kV) + {}_kV$.

153) ${}_t\overline{L}(\overline{A}_x) = v^{T_{x+t}} - \overline{P}(\overline{A}_x)\overline{a}_{\overline{T_{x+t}|}} = \overline{Z}_{x+t} - \overline{P}(\overline{A}_x)\overline{Y}_{x+t}$. where $T_{x+t} = T_x - t$. Taking expectations gives ${}_t\overline{V}(\overline{A}_x) = \overline{A}_{x+t} - \overline{P}(\overline{A}_x)\overline{a}_{x+t}$.

154) A *gross premium* G takes into account expenses. Then an expense–augmented reserve = *gross premium reserve* ${}_tV^E$ = (APV of future benefits and expenses) − (APV of future gross premiums).

155) Suppose premiums P are paid every year including at time t. Let $0 \le s \le 1$, then ${}_{t+s}V \approx ({}_tV + P)(1 - s) + ({}_{t+1}V)s$. Note that $({}_tV + P)$ is the terminal reserve at time $t+$ just after the premium P has been paid. Also ${}_0V = 0$ under the equivalence principle.

156) Consider a fully continuous whole life model with payment benefit b_r at time of death r and benefit payment at rate $\overline{P}(r)$ at time r. Provided $T_x > t$, the NLP terminal reserve is ${}_tV = \int_0^\infty b_{t+s}\ e^{-\delta s}\ {}_sp_{x+t}\ \ \mu_{x+t+s}\ ds - \int_0^\infty \overline{P}(t+s)\ e^{-\delta s}\ {}_sp_{x+t}\ ds$.

157) **Know** In 156), suppose $\overline{P}(t+s) = \pi_0 e^{\gamma(t+s)}$ for $s, t \ge 0$, $\mu_{x+t} \equiv \mu$ for $t > 0$ so $T_{x+t} \sim EXP(\mu)$ for any $t \ge 0$, $b_{t+s} = Je^{\theta(t+s)}$ for $s, t \ge 0$. Then ${}_sp_{x+t} = e^{-\mu s}$, for $s, t \ge 0$. Then for $T_x > t$, $\pi_0 = \dfrac{J\mu(\mu + \delta - \gamma)}{\mu + \delta - \theta}$, and ${}_t\overline{V} = \dfrac{J\mu\ e^{\theta t}}{\mu + \delta - \theta} - \dfrac{\pi_0\ e^{\gamma t}}{\mu + \delta - \gamma}$.

158) **Know** In 157), $\overline{P}(t+s) = \pi_0 e^{\gamma(t+s)}$ is often written as *the annual premium rate is* $\pi_0 e^{\gamma t}$ for all t (or $t \geq 0$). The benefit $b_{t+s} = Je^{\theta(t+s)}$ is often written as *the benefit is* $Je^{\theta t}$ *if death occurs at time* t. If $\gamma = 0$, then $\overline{P}(t+s) \equiv \pi_0 = \dfrac{J\mu(\mu+\delta)}{\mu+\delta-\theta}$ for $s, t \geq 0$. If $\theta = 0$, then $b_{t+s} \equiv J$ for $s, t \geq 0$.

More Topics from Ch. 7, 8, 9, 10

ch. 9

159) **Know:** Assume time and cause of decrement are independent. Suppose there are m decrements and a continuous whole life insurance pays benefit $b_t^{(j)}$ if decrement j occurs at time t. Let $\overline{Z}$ be the benefit random variable for the insurance. Then the **single benefit premium** (buy the insurance at time 0 with 1 payment, also called the net single premium) is

$$\text{APV} = \overline{A} = E[\overline{Z}] = \sum_{j=1}^{m} \int_0^\infty b_t^{(j)} e^{-\delta t} \; {}_tp_x^{(\tau)} \mu_{x+t}^{(j)} \, dt = \sum_{j=1}^{m} \overline{A}^{(j)}, \text{ and}$$

$$E[\overline{Z}^2] = \sum_{j=1}^{m} \int_0^\infty [b_t^{(j)}]^2 e^{-2\delta t} \; {}_tp_x^{(\tau)} \mu_{x+t}^{(j)} \, dt = \sum_{j=1}^{m} {}^2\overline{A}^{(j)}.$$

If $\mu_{x+t}^{(j)} \equiv \mu^{(j)} \equiv \mu_j$, and $b_t^{(j)} \equiv b_j$ are free of $t \geq 0$, then $\mu^{(\tau)} = \sum_{j=1}^m \mu^{(j)}$, the single benefit premium $= E[\overline{Z}] = \displaystyle\sum_{j=1}^{m} \frac{b_j \mu^{(j)}}{\mu^{(\tau)} + \delta}$, and $E[\overline{Z}^2] = \displaystyle\sum_{j=1}^{m} \frac{(b_j)^2 \; \mu^{(j)}}{\mu^{(\tau)} + 2\delta}$.

160) For discrete whole life insurance as in 159) except the benefit b_j is paid at the end of the year $k = 1, 2, ...,$ if decrement j occurs in the $(k-1)$th year, $\overline{A}^{(j)} = b_j \; \sum_{k=0}^\infty v^{k+1} \;\; {}_kp_x^{(\tau)} \; q_{x+k}^{(j)}$, and APV = single net premium $= \sum_{j=1}^m \overline{A}^{(j)}$.

161) If a contract is taken out that pays b_j if decrement j occurs but 0 otherwise, then the APV of the contract is APV = single net premium $= \overline{A}^{(j)}$. (Just set $b_k = 0$ for the other decrements.)

ch. 10

162) Premiums for joint life status or last survivor status are calculated in the usual way under the equivalence principle. Let (w) = (xy) or (w) = $(\overline{xy})$. Then APV insurance = APV premiums so $P = \dfrac{A_w}{a_w}$.

163) **Know:** For fully continuous whole life insurance (with premiums paid continuously until first death for (xy) and until second death for $(\overline{xy})$), $\overline{P}(\overline{A}_{xy}) = \dfrac{\overline{A}_{xy}}{\overline{a}_{xy}}$ for the joint life status, while $\overline{P}(\overline{A}_{\overline{xy}}) = \dfrac{\overline{A}_{\overline{xy}}}{\overline{a}_{\overline{xy}}} = \dfrac{\overline{A}_x + \overline{A}_y - \overline{A}_{xy}}{\overline{a}_x + \overline{a}_y - \overline{a}_{xy}}$ for the last survivor status. These formulas are for unit benefit.

164) Variant: if premiums are paid until first death but insurance is paid after last death, then $P = \dfrac{A_{\overline{xy}}}{a_{xy}}$. See 165).

165) **Know:** Fully continuous whole life insurance of 1 on the last survivor of (x) and (y), but premiums payable (continuously) until first death has premium $\overline{P} = \dfrac{\overline{A}_{\overline{xy}}}{\overline{a}_{xy}} =$

$\dfrac{\overline{A}_x + \overline{A}_y - \overline{A}_{xy}}{\overline{a}_{xy}}$.

166) Often have $T_x \sim EXP(\mu_1) \perp\!\!\!\perp T_y \sim EXP(\mu_2)$. Then $T_{xy} \sim EXP(\mu_1 + \mu_2)$, $\overline{A}_w \stackrel{E}{=} \dfrac{\mu}{\mu + \delta}$, and $\overline{a}_w \stackrel{E}{=} \dfrac{1}{\mu + \delta}$ where w and μ correspond to x, y, or xy.

A variant is the common shock model where $T_x \sim EXP(\mu_x = \mu_1^* + \lambda), T_y \sim EXP(\mu_y = \mu_2^* + \lambda)$, and $T_{xy} \sim EXP(\mu_1^* + \mu_2^* + \lambda)$.

167) The terminal reserve for a joint life status (w) = (xy) is like the reserve for a single life status (w), provided that the status (w) = (xy) has not yet failed at time t. Note that the subscript $w + t = x + t : y + t$.

168) The terminal reserve for a last survivor life status (w) = ($\overline{xy}$) is more complicated, but is like the reserve for a single life status (w), provided that both (x) and (y) still survive at time t. Then the subscript $w + t = \overline{x + t : y + t}$.

∮ 6.4.2 **gross (annual) premium** = contract (annual) premium

169) For a gross premium G, the equivalence principle says that $E({}_0L_e) = E({}_0L) =$ APV(benefits + expenses) − APV(gross premiums), where ${}_0L_e = {}_0L$ is the loss random variable at issue. One type of problem uses this equation to solve for G.

170) A variant of 169) is to find (the observed value of) ${}_0L_e =$ ${}_0L =$ APV(benefits + expenses) − APV(gross premiums) for person who died in the kth year where k is small and G is given.

∮ 7.2.4

171) $${}_kAS = \frac{[\,{}_{k-1}AS + G(1 - c_{k-1}) - e_{k-1}](1+i) - b_k q_{x+k-1}^{(d)} - {}_kCV q_{x+k-1}^{(w)}}{1 - q_{x+k-1}^{(d)} - q_{x+k-1}^{(w)}}$$

172) Given all of the variables in 171) except one, usually ${}_kAS$ or i, calculate the unknown variable where ${}_kAS$ is the asset share at the end of year k,
G is the gross premium (= contract premium),
c_k is the proportion of the premium payable as an expense at time k, starting at $k = 0$,
e_k is the per policy expense at time k,
b_k is the face amount,
$q^{(d)} = q^{(1)}$ is the death probability,
$q^{(w)} = q^{(2)}$ is the withdrawal probability,
${}_kCV$ is the cash value at time k.

173) Usually assume ${}_0AS = 0$.

174) The formula is 171) is for fully discrete insurance. So premiums are paid at the beginning of the year and benefits at the end of the year.