The final is Tue. Dec. 10. **You are allowed 16 sheets of notes and a calculator.**
Need to know Exam 1, 2 and 3 reviews. This is material since Exam 3.

117) The GARCH(p_g, q_g) model is stationary if $\sum_{i=1}^m (\alpha_i + \beta_i) < 1$ where $m = \max(p_g, q_g)$, assuming the $\alpha_i > 0$, $\beta_i > 0$ constraints are used.

118) Let $\hat{\epsilon}_t = r_t / \hat{\sigma}_{t|t-1}$ be the standardized residuals. The ACF and PACF of $\hat{\epsilon}_t$, $|\hat{\epsilon}_t|$ and $\hat{\epsilon}_t^2$ should resemble those of a white noise. The generalized Portmanteau test is a modified Ljing Box test on $|\hat{\epsilon}_t|$ or $\hat{\epsilon}_t^2$. Want the plotted points to lie above the 0.05 horizontal line.

119) Have been doing time domain time series. Frequency domain time series uses spectral analysis which models a time series with a linear combination of cosine curves where a cosine curve is $R\cos(2\pi f t + \Phi) = A\cos(2\pi f t) + B\sin(2\pi f t)$. Here $R > 0$ is the amplitude, f = frequency, Φ = phase, $1/f$ is the period of the curve, $R = A^2 + B^2$, $A = R\cos(\Phi)$, $B = -R\sin(\Phi)$, and $\Phi = \arctan(-B/A)$.

120) A time series model is $Y_t = A_0 + \sum_{j=1}^m [A_j \cos(2\pi f_j t) + B_j \sin(2\pi f_j t)] + e_t$. This model can be fit with least squares, using

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & \cos(2\pi f_1 t_1) & \sin(2\pi f_1 t_1) & \dots & \cos(2\pi f_m t_1) & \sin(2\pi f_m t_1) \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ 1 & \cos(2\pi f_1 t_n) & \sin(2\pi f_1 t_n) & \dots & \cos(2\pi f_m t_n) & \sin(2\pi f_m t_n) \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ B_1 \\ \vdots \\ A_m \\ B_m \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}.$$

Output in symbols and for a data set are shown below.

coefficients	Estimate	Std. Err,	t value	p-value= $Pr(>\|t\|)$
intercept	$\hat{A}_0$	$se(\hat{A}_0)$	$t = \hat{A}_0/se(\hat{A}_0)$	for Ho: $A_0 = 0$
cos1	$\hat{A}_1$	$se(\hat{A}_1)$	$t = \hat{A}_1/se(\hat{A}_1)$	for Ho: $A_1 = 0$
sin1	$\hat{B}_1$	$se(\hat{B}_1)$	$t = \hat{B}_1/se(\hat{B}_1)$	for Ho: $B_1 = 0$
	$\vdots$			
cosm	$\hat{A}_m$	$se(\hat{A}_m)$	$t = \hat{A}_m/se(\hat{A}_m)$	for Ho: $A_m = 0$
sinm	$\hat{B}_M$	$se(\hat{B}_m)$	$t = \hat{B}_m/se(\hat{B}_m)$	for Ho: $B_m = 0$

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Coefficients Estimate Std. Error t value Pr(>|t|)
(Intercept)  48.9426     0.1823  268.45   <2e-16 ***
cos1         -9.3016     0.2578  -36.08   <2e-16 ***
sin1         -6.9901     0.2578  -27.11   <2e-16 ***
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121) **Know for final:** Let τ_k be A_k or B_k. The 4 step test of hypotheses for $Ho: \tau_k = 0$ is i) State the hypotheses Ho: $\tau_k = 0$ Ha: $\tau_k \neq 0$.
ii) Find the test statistic t from output.
iii) Find pval = the estimated p–value from output.
iv) State whether you reject Ho or fail to reject Ho and give a conclusion. Reject Ho if pval $\leq \alpha$ and fail to reject Ho if pval $> \alpha$. **Use $\alpha = 0.05$ if α is not given.** For $\tau_k = A_k$

(B_k), conclude $\cos(2\pi f_k t)$ ($\sin(2\pi f_k t)$) is needed in the model if Ho is rejected. Conclude $\cos(2\pi f_k t)$ ($\sin(2\pi f_k t)$) is not needed in the model given the other terms are in the model if you fail to reject Ho. If $\tau_k = A_0$, the intercept is needed in the model if Ho is rejected. The intercept is not needed in the model if you fail to reject Ho.

122) The Fourier frequencies are $1/n, 2/n, ..., k/n$ where $k/n = (n-1)/(2n)$ for odd $n = 2k+1$ and $k/n = (n/2)/n = 1/2$ for even $n = 2k$. The periodogram $I(f)$ is $I(j/n) = \frac{n}{2}(\hat{A}_j^2 + \hat{B}_j^2)$ for $j = 1, ..., k$ for odd n and for $j = 1, ..., k-1$ for even n where $I(1/2) = n(\hat{A}_k)^2$. For time series model 120), the periodogram will show m spikes at frequencies f_j if these frequencies are Fourier freqeuncies. This will sometimes happen for quarterly or monthly seasonal time series provided that all 4 quarters or 12 months of each year are observed.

123) For model 120), the fitted values $\hat{Y}_t$ and residuals $\hat{e}_t$ may only take on several distinct values if $m = 1$. The plotted points in the response plot should follow the ideitity line and the plotted points in the residual plot should follow the horizontal line at 0. The ACF and PACF of the residuals should resemble those of a white noise. The McLeod Li plot of the residuals should have the plotted points above the 0.05 horizontal line. See HW 11C).

124) The SETAR(d, p_1, p_2) model has $Y_t = \phi_{1,0} + \phi_{1,1} Y_{t-1} + \cdots + \phi_{1,p_1} Y_{t-p_1} + e_t$ if $Y_{t-d} \leq r$ (the AR(p_1) model for the lower regime), and $Y_t = \phi_{2,0} + \phi_{2,1} Y_{t-1} + \cdots + \phi_{2,p_2} Y_{t-p_2} + e_t$ if $Y_{t-d} > r$ (the AR(p_2) model for the upper regime) where $\{e_t\}$ is a white noise.

125) Points in the lower (upper) regime are used to fit the AR model for the lower (upper) regime, resulting in the output below.

coefficients	Estimate	Std. Err.	t value	p-value= $Pr(>\|t\|)$
		lower	regime	
intercept-Y	$\hat{\phi}_{1,0}$	$se(\hat{\phi}_{1,0})$	$t = \hat{\phi}_{1,0}/se(\hat{\phi}_{1,0})$	for Ho: $\phi_{1,0} = 0$
lag1-Y	$\hat{\phi}_{1,1}$	$se(\hat{\phi}_{1,1})$	$t = \hat{\phi}_{1,1}/se(\hat{\phi}_{1,1})$	for Ho: $\phi_{1,1} = 0$
	$\vdots$			
lagp_1-Y	$\hat{\phi}_{1,p_1}$	$se(\hat{\phi}_{1,p_1})$	$t = \hat{\phi}_{1,p_1}/se(\hat{\phi}_{1,p_1})$	for Ho: $\phi_{1,p_1} = 0$
		upper	regime	
intercept-Y	$\hat{\phi}_{2,0}$	$se(\hat{\phi}_{2,0})$	$t = \hat{\phi}_{2,0}/se(\hat{\phi}_{2,0})$	for Ho: $\phi_{2,0} = 0$
lag1-Y	$\hat{\phi}_{2,1}$	$se(\hat{\phi}_{2,1})$	$t = \hat{\phi}_{2,1}/se(\hat{\phi}_{2,1})$	for Ho: $\phi_{2,1} = 0$
	$\vdots$			
lagp_1-Y	$\hat{\phi}_{2,p_2}$	$se(\hat{\phi}_{2,p_2})$	$t = \hat{\phi}_{2,p_2}/se(\hat{\phi}_{2,p_2})$	for Ho: $\phi_{2,p_2} = 0$

121) **Know for final:** Let τ_k be $\phi_{1,k}$ or $\phi_{2,k}$. The 4 step test of hypotheses for $Ho: \tau_k = 0$ is i) State the hypotheses Ho: $\tau_k = 0$ Ha: $\tau_k \neq 0$.
ii) Find the test statistic t from output.
iii) Find pval = the estimated p–value from output.
iv) State whether you reject Ho or fail to reject Ho and give a conclusion. Reject Ho if pval $\leq \alpha$ and fail to reject Ho if pval $> \alpha$. **Use** $\alpha = 0.05$ **if** α **is not given.** Conclude Y_{t-k} is needed in the model for the lower or upper regime if Ho is rejected. Conclude Y_{t-k} is not needed in the model for the lower or upper regime given the other terms are in the model if you fail to reject Ho. The $k = 0$ tests are for the two intercepts.