

HW 6 is due on Friday, Oct. 4. Quiz 5 covers probability, the Law of Large Numbers and using the normal curve for \bar{x} (including this HW and last HW, so Exam 2 review 26)-41)). (Exam 2 is Wednesday, Oct. 16.) **Two pages: problems A)- L)**

A) 10.11

outcome	1	2	3	4	5	6
model 1	1/7	1/7	1/7	1/7	1/7	1/7
model 2	1/3	1/6	1/6	0	1/6	1/6
model 3	1/3	1/6	1/6	1/6	1/6	1/6
model 4	1	1	2	1	1	2

Above are some models for rolling a die. Some models are not legitimate because they do not obey the rules of probability. Which models are legitimate and are not? For the illegitimate models, explain what is wrong.

comment: Only 1 model is legitimate. See box on p. 271. The point of model 2 is that not all die are fair ($P(i) = 1/6$ for $i = 1, \dots, 6$), but biased die can still be described by probability theory.

B) The 2000 census gave these probabilities:

	Asian	Black	White	Other
Hispanic	0.000	0.003	0.060	0.062
Not Hispanic	0.036	0.121	0.691	0.027

- What is the probability that a randomly chosen American is Hispanic?
- What is the probability that a randomly chosen American is not a member of the Non-Hispanic White group (the majority group)?

comment for B) and C): See p. 269. The outcomes in the table are disjoint.

C) M&Ms are a candy with the distribution given below.

color	Purple	Yellow	Red	Orange	Brown	Green	Blue
probability	0.2	0.2	0.2	0.1	0.1	0.1	?

- What is the probability of Blue?
- What is the probability of not drawing a brown candy?
- What is the probability that the candy you draw is either yellow, orange or red?

D) 11.5 Suppose that the mean loss from a fire in a year is \$250 a year. (Most of us have no loss, but a few lose their homes. The \$250 is the average loss.) An insurance company plans to sell fire insurance for \$250 plus enough to cover costs and profit. Explain clearly why it would be unwise to sell only 12 policies. Then explain why selling thousands of such policies is safe business.

comment: See p. 294 and ex. 11.3. (Law of Large Numbers)

E) Suppose that a 12th grade assessment score is approximately normal with mean $\mu = 300$ and standard deviation $\sigma = 35$. a) What is the probability that a randomly selected student has a score i) higher than 300? ii) Higher than 335?

b) Now choose a SRS of four 12th graders. What is the probability that the mean score is i) higher than 300? ii) Higher than 335?

comment: Part a) is a forwards calculation for x , see p. 81-83. Part b) is a forwards calculation for \bar{x} with $n = 4$. See p. 299, 302, and ex 11.8 on p. 305. (Normal)

F) The 2000 ACT scores were approximately normal with mean $\mu = 21.0$ and standard deviation $\sigma = 4.7$. a) What is the probability that a randomly selected score is 23 or higher? (Answer near 0.33.)

b) Now take a SRS of 50 students scores. i) What are the mean and standard deviation of the sample mean score \bar{x} of these 50 students? ii) What is the probability that the mean score \bar{x} is 23 or higher? (Answer near 0.001.)

comment: See 299, 302, and ex 11.8 on p. 305. (Normal) In a) $n = 1$ but in b) $n = 50$.

G) 11.24 A roulette wheel has 38 slots of which 18 are red. A bet of \$1 on red returns \$2 if the ball lands in a red slot, otherwise the gambler loses her \$1. The mean payoff per \$1 bet is 94.7 cents. Explain what the law of large numbers tells about what will happen if a gambler makes very many bets on red.

comment: The payoff is 94.7 cents per \$1 bet, so the gambler will lose 5.3 cents per \$1 bet. Also see p. 294 - 295. (Law of large numbers.)

H) In the long run, annual returns on common stocks have mean 13 and standard deviation 17 (in percent). What is the probability that the mean annual return in the next 45 years will i) exceed 15? ii) Be less than 10?

comment: See 299, 302, and ex 11.8 on p. 305. (Normal) Notice that $n = 45$. Assume this n is large enough for the CLT to hold. The answers are near 0.21 and 0.12.

I) Suppose that the color of cars that pass by are independent, $P(\text{black}) = 0.112$ and $P(\text{white}) = 0.156$. Suppose two cars pass by. a) Find the probability both are black. b) Find the probability that the first is black and the 2nd white.

comment: See p. 317. By independence, you may use the multiplication rule.

J) A string of 20 holiday lights is wired in series so that if any light fails, the whole string will go dark. Each light has a probability of 0.02 of failing during the holiday season. The lights fail independently of each other. What is the probability that the string of lights will remain bright?

comment for J) and K): See ex. 12.3 and 12.4. The answers are near 0.66 and 0.001.

K) Suppose a slot machine has 3 wheels each with 20 symbols that are equally likely to show when the wheels stop spinning. The three wheels are independent of each other, the middle wheel has nine bells while the left and right wheels have 1 bell each. You win the jackpot if all three wheels show bells. What is the probability of winning the jackpot?

L) Suppose $P(A) = 0.138$, $P(B) = 0.261$ and $P(A \text{ and } B) = 0.082$. i) Draw a Venn diagram that shows the relationship between the events A and B. ii) Find $P(A \text{ or } B)$.

comment: See ex. 12.5. The answer is near 0.32.