

Math 401 HW 3 Fall 2024. Due Friday, Sept. 13.
Quiz 3 Wed. Sept. 11 is like HW 2–3.

Problem numbers, like 5.6ab, are actually from Camilli, S.J., Duncan, I., and London, R.L. (2014), *Models for Quantifying Risk*, 6th ed. ACTEX Publications, Winsted, CT.

1) 5.6ab Let X_1 and X_2 be independent random variables. Define the variables $Y = \min(X_1, X_2)$ and $Z = \max(X_1, X_2)$.

- a) Show $S_Y(y)$ is the product of the SDF's of X_1 and X_2 .
- b) Show that $F_Z(z)$ is the product of the CDF's of X_1 and X_2 .

2) 5.9: Given that $\lambda_0(t) = kt^n$ and $\lambda_0(22) = 1.26$, where 22 is the median age of the distribution, find the value of n .

Hint: this is the Weibull distribution with $S_0(t) = \exp\left[\frac{-kt^{n+1}}{n+1}\right]$. Now $\lambda_0(22) = 1.26 = k(22)^n$ and $\log[S_0(22)] = \log(0.5) = -\log(2)$ is a function of $k(22)^n$ and n . Solve for n .

3) Let $T_0 \sim U(0, \omega)$ where $\omega > 0$. Let $0 < x < \omega$. Find the distribution of T_x by finding the pdf of T_x . Find the pdf $f_x(t)$ using $f_x(t) = f_0(x+t)/S_0(x)$.

Hint: The support of a continuous random variable Y is the set of values where the pdf of Y is positive. Given $T_0 > x$, we have that $x + T_x = T_0$. The support of T_0 is $(0, \omega)$. Hence the support of T_x is $(0, \omega - x)$. Find the pdf $f_x(t)$ of T_x for $0 < t < \omega - x$.

4) Suppose $S_0(x) = [1 - (x/100)]^{1/2}$ for $0 \leq x \leq 100$.

a) Find ${}_{17}p_{19}$.

b) Find ${}_{15}q_{36}$.

c) Find $\mu_0(36) = \mu_{36}$.

5) Let $\mu(t) = \mu_{x+t}$. Let $R = 1 - p = q = 1 - \exp\left[-\int_0^1 \mu(t) dt\right] = 1 - \exp\left[-\int_0^1 \mu_{x+t} dt\right]$.

Let $S = 1 - \exp\left[-\int_0^1 (\mu(t) + k) dt\right] = 1 - \exp\left[-\int_0^1 (\mu_{x+t} + k) dt\right] = 0.75R$. Find k as a function of q .

Hint: This is basically problem 59 on the SOA MLC practice problems.

(<https://www.soa.org/4934f7/globalassets/assets/files/edu/edu-2014-spring-mlc-ques.pdf>) has the SOA MLC practice problems and (<https://www.soa.org/globalassets/assets/files/edu/edu-2014-spring-mlc-sol.pdf>) has the solutions. Google “soa mlc sample questions”

(<https://www.casact.org/exams-admissions/exam-results-summary-exam-statistics/past-exams-pass-marks>) and (<https://www.soa.org/education/exam-req/syllabus-study-materials/edu-multiple-choice-exam/>) have some old professional exams. Google “cas exam 3L”

I also have old MLC questions and solutions if you bring a flash drive.