Math 402 HW 10 Spring 2023. Due Friday, April 21. Final: Friday, May 12, 10:15-12:15

$$\boldsymbol{P} = \begin{bmatrix} & F & G & H & I \\ F & 0.2 & 0.8 & 0.0 & 0.0 \\ G & 0.5 & 0.0 & 0.5 & 0.0 \\ H & 0.75 & 0.0 & 0.0 & 0.25 \\ I & 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}.$$

1) A machine is in one of four states (F, G, H, I) and migrates annually among them according to a Markov chain with the above transition matrix. At time 0, the machine is in state F.

a) Find the probability that the machine is in state F at the end of 3 years.

b) If a salvage company pays 500 at the end of three years if the machine is in state F, then the APV at time 0 for the contract $= 500\ v^3$ [probability from a)]. Calculate the APV if $v = 0.9$.

2) Suppose $T_x \sim EXP(0.02) \perp\!\!\!\perp T_y \sim EXP(0.03)$ and $\delta = 0.01$. Find $\overline{a}_{x|y}$.

3) A policy with a gross premium of 1000 and survival model $q_{x+t} = 0.05 + 0.01t$ for $t = 0, 1, 2, 3$ produces profit signature $\mathbf{\Pi} = (-300, 100, 90, 80, 70)$. The hurdle rate $r = 0.1$.

a) Calculate the NPV of the profits at issue.

b) Show that the internal rate of return is 5.54%. Hint: show "NPV = 0" acting as if $r = 0.0554$. Take $NPV \approx 0$ if $|NPV| \leq 0.05$.

c) Find the DPP (discounted payback period) if it exists. Hint: the DPP does not exist if $NPV(t) < 0$ for $t = 0, 1, ..., n$. Otherwise, $DPP = m$ where m is the first time $NPV(t) \geq 0$.

d) Calculate the profit margin.

Hint: APV (gross premium) $= 1000\left(1 + \frac{p_x}{1+r} + \frac{{}_2p_x}{(1+r)^2} + \frac{{}_3p_x}{(1+r)^3}\right) = 1000\ddot{a}_{x:\overline{n}|} = 1000\sum_{k=0}^{n-1} v_r^k\ {}_kp_x$ where ${}_kp_x = \prod_{t=0}^{k-1}(1 - q_{x+t}) = \prod_{t=0}^{k-1} p_{x+t}$. See 189) of Exam 3 review.

t	Pr_t	${}_tp_x$	Π_t	$NPV(t) = \sum_{k=0}^{t} \Pi_k v_r^k$
0	−5000.00	1	−5000.00	
1	3838.20	0.98500	3838.20	
2	3018.70	0.96826	2973.42	
3	1635.70	0.94986	1583.78	
4	1684.70	0.92991	1600.23	
5	1503.71	0.90759	1398.32	

4) a) For the above table with $r = 0.1$, find NPV(t), the DPP (if it exists), and NPV (give the last column of the above table).

b) If the gross premium $G = 20335$, show the profit margin ≈ 0.05. Hint: APV (gross premium) $= G\sum_{t=0}^{n-1} v_r^t\ {}_tp_x$ where ${}_tp_x$ is given in the above table.