Math 402 HW3 Spring 2023. Due Friday, Feb. 10.

1) A generalized DeMoivre $GD(\alpha, \omega)$ distribution has survival function $S_0(x) = \left(\frac{\omega - x}{\omega}\right)^\alpha$ for $0 < x < \omega$ where $\alpha > 0$. Then $T_x \sim GD(\alpha, \omega - x)$ where $0 < x < \omega$ is fixed. For $0 < t < \omega - x$, find the following quantities. Exam 1 review 63) has the final answers.

a) $S_x(t) = {}_tp_x$

b) $F_x(t) = {}_tq_x$

c) $f_x(t) = {}_tp_x \ \mu_{x+t}$

d) $\mu_x(t) = \mu_{x+t}$

e) $E(T_x) = \overset{o}{e}_x = \int_0^{\omega - x} {}_tp_x \ dt$

2) Suppose (x) follows a DeMoivre distribution with ω_x and (y) follows a DeMoivre distribution with ω_y so $T_x \sim U(0, \omega_x - x) \perp\!\!\!\perp T_y \sim U(0, \omega_y - y)$. Let $a = \min(\omega_x - x, \omega_y - y)$ and $b = \max(\omega_x - x, \omega_y - y)$. Then $\overset{o}{e}_{xy} = \frac{a}{2} - \frac{a^2}{6b}$ and $\overset{o}{e}_{\overline{xy}} = \frac{b}{2} + \frac{a^2}{6b}$. Suppose $(40) \perp\!\!\!\perp (55)$, (40) follows a DeMoivre distribution with $\omega_x = 100$ and (55) follows a DeMoivre distribution with $\omega_x = 105$. In class showed $\overset{o}{e}_{40:55} = 325/18$. Find $\overset{o}{e}_{\overline{40:55}}$.

3) You are given T_x and T_y are independent, the survival function for (x) follows DeMoivre's law with $\omega = 100$, the survival function for (y) is based on a constant force of mortality $\mu_{y+t} = \mu$ for $t \geq 0$ and $n < 100 - x$. Determine the probability that (x) dies before (y) and within n years. Hint: the answer depends on n and μ. Also ${}_tp_x = \frac{100 - x - t}{100 - x} \quad \left(= \frac{\omega - x - t}{\omega - x}\right)$ and $\mu_{x+t} = \frac{1}{100 - x - t} \quad \left(= \frac{1}{\omega - x - t}\right)$. Want ${}_nq^1_{xy} = \int_0^n {}_tp_{xy} \ \mu_{x+t} \ dt = \int_0^n {}_tp_x \ {}_tp_y \ \mu_{x+t} \ dt.$

4) You are given (x) and (y) are independent lives, $\mu_{x+t} = 5t$ for $t \geq 0$ is the force of mortality for (x), and $\mu_{y+t} = t$ for $t \geq 0$ is the force of mortality for (y).

a) Find ${}_tp_x = \exp(-\int_0^t \mu_{x+s} ds)$.

b) Find ${}_tp_y = \exp(-\int_0^t \mu_{y+s} ds)$.

c) Find q^1_{xy}.

Hint: $q^1_{xy} = {}_1q^1_{xy} = \int_0^1 {}_tp_{xy} \ \mu_{x+t} \ dt = \int_0^1 {}_tp_x \ {}_tp_y \ \mu_{x+t} \ dt.$

5) Suppose $T_x \sim EXP(0.01) \perp\!\!\!\perp T_y \sim EXP(0.02)$ and $\delta = 0.03$.

a) Find $\overline{A}_{xy}$.

b) Find ${}^2\overline{A}_{xy}$.

c) Find $\overline{a}_{xy}$.

d) Find $V[\overline{Y}_{xy}]$.

Hint: See E1 review 72).

Problems 1, 12, 26, 31, 46, 49, 53, 57, 80, 91, 94, 104, 122, 123, 128, 150, 163, 189, 191, 193, 194, 225, 233, 249, 261, 262, 263, 265, 266, and 269-273 from the SOA MLC practice exam may cover ch. 10 topics.