Math 402 HW 5 Spring 2023. Due Friday, March 3. Quiz 5 Wed. March 1.

1) Suppose that for a life insurance contract on (x), decrements are due to death (cause 1) and withdrawal (cause 2). Assume

i) $\mu_x^{(1)}(t) = 0.01, \quad t > 0,$

ii) $\mu_x^{(2)}(t) = 0.05, \quad t > 0.$

Calculate the probability of a policy withdrawal in the first ten years.

(Hint: want $\ _{10}q_x^{(2)}$. See exam 1 review 85 i).)

2) The probability that someone succumbed (became inactive) due to cause j at time t, given that person succumbed (to some decrement) at time t is $\dfrac{\mu_{x+t}^{(j)}}{\mu_{x+t}^{(\tau)}}$.

Find the above probability for $j = 1$ in the triple decrement model if $\mu_{x+t}^{(1)} = 0.03$, $\mu_{x+t}^{(2)} = 0.03t$, and $\mu_{x+t}^{(3)} = 0.03t^2$ for $t > 0$.

3) For a double decrement table, you are given $\mu_{x+t}^{(1)} = 0.2\mu_{x+t}^{(\tau)}$ for $t > 0$, $\mu_{x+t}^{(\tau)} = kt^2$ for $t > 0$, and $q_x^{'(1)} = 0.04$. Calculate $\ _2q_x^{(2)}$.

Hint: MLC problem 20: $\mu_{x+t}^{(2)} = 0.8\mu_{x+t}^{(\tau)}$ for $t > 0$. Use $q_x^{'(1)} = \ _1q_x^{'(1)} = 1 - p_x^{'(1)} = 1 - \ _1p_x^{'(1)} = 1 - \exp[-\int_0^1 \mu_{x+s}^{(1)}\, ds]$ to find k (near 0.6). Then use $\ _2q_x^{(2)} = \int_0^2 \ _sp_x^{(\tau)}\mu_{x+s}^{(2)} ds = 0.8\int_0^2 \ _sp_x^{(\tau)}\mu_{x+s}^{(\tau)} ds = 0.8\ _2q_x^{(\tau)} = 0.8[1 - \ _2p_x^{(\tau)}]$ where $\ _2p_x^{(\tau)} = \exp[-\int_0^2 \mu_{x+s}^{(\tau)}\, ds]$.

4) The APV for a last-survivor whole life insurance on $(\overline{xy})$ with unit benefit paid at the instant of failure of the status, was calculated assuming independent future lifetimes for (x) and (y) with constant hazard rate 0.06 for each. It is now discovered that although the total hazard rate of 0.06 is correct, the two lifetimes are not independent since each includes a common shock hazard factor with constant force 0.02. The force of interest used in the calculation is $\delta = 0.05$. Calculate the increase in the APV that results from recognition of the common shock element.

Hint: for the original incorrect model, $T_{xy} \sim EXP(0.06 + 0.06)$. For the correct common shock model, $T_{xy} \sim EXP(0.04 + 0.04 + 0.02)$ since $\mu_x = \mu_y = 0.06 = \mu_x^* + 0.02 = \mu_y^* + 0.02$.

5) For the common shock model with $\lambda = 0.01$, assume that the noncommon force of mortality for (x) is 0.02, and the noncommon force of mortality for (y) is 0.03. Calculate the expected lifetime of the last survivor status.

Hint: The term "noncommon" means you are given μ_x^* and μ_y^*. Need to find $\overset{o}{e}_{\overline{xy}}$.

6) Suppose 1000 white 71 year old females buy a 1 year \$100000 life insurance policy. Actuaries use $1 - S(t+a)/S(a) = 1 - P(Y > t + a | Y > a)$ to estimate how many claims will be filed. Hence actuaries want $S(72)/S(71)$. If $\hat{S}(72) = 0.77$ and $\hat{S}(71) = 0.78$, about how many of the 1000 claims will be filed?

7) Survival times in days after being inoculated with human tuberculosis for seven mice are 41, 44, 46, 54, 55, 58, 60.

a) Find $\hat{S}_E(54)$.

b) Find a 95% CI for $S(54)$.