

Math 402 HW 6 Spring 2023. Due Friday, March 10. Quiz 6 Wed. March 8. Exam 2 Wednesday March 29. Cumulative Final: Friday, May 12, 10:15-12:15.

1) Times (in years) until death are given for 9 people from Dickson, Mary, and Waters (problem 18.1 modified, 2020, p. 690).  
27, 30+, 34, 58+, 68, 68+, 70, 77, 78+

a) Compute the Kaplan Meier survival function  $\hat{S}_K(t_i)$  by making a table of the data with headers as done in class.

b) Use Greenwood's formula to find  $SE(\hat{S}_K(69))$ , an estimate of the standard deviation of  $\hat{S}_K(69)$ .

2) Suppose 10000 lives are observed from exact age 50 for 30 years, giving the table below.

| age last birthday | deaths |
|-------------------|--------|
| 50–59             | 170    |
| 60–69             | 465    |
| 70–74             | 552    |
| 75–79             | 968    |

a) Find  $\hat{S}_{50}(t)$  for the four possible values of  $t$  using a table with headers as done in class.

b) Find the ogive empirical survival function at  $t = 28$ .

3)

| $j$ | $t_j$ | $d_j$ | $r_j$ | $\hat{S}_K(t_j)$ |
|-----|-------|-------|-------|------------------|
| 1   | 1     | 3     | 15    |                  |
| 2   | 2     | 24    | 80    |                  |
| 3   | 3     | 5     | 25    |                  |
| 4   | 4     | 6     | 60    |                  |
| 5   | 5     | 3     | 10    |                  |

a) Fill in the above table on a separate page, where  $\hat{S}_K(t_j)$  is the Kaplan Meier product limit estimator.

b) Find  $\hat{H}_N(5)$ .

c) Use b) to find  $\hat{S}_N(5)$ .

d) Find  $\hat{S}_K(5)$ .

e) Determine the Greenwood approximation for the variance of the Kaplan Meier product limit estimator  $\hat{S}_K(4)$ .

Hint: See old quiz 6. **Over for more problems.**

4) For a discrete whole life insurance of 150,000 on  $x$ , you are given  ${}^2A_x = 0.0143$ ,  $A_x = 0.0653$ , and the annual premium is calculated using the equivalence principle. Calculate the standard deviation of the loss variable  $L$ .

Hint: For unit payment, say  $V[L_x(1)]$ ,

$$V(L_x(1)) = \frac{{}^2A_x - (A_x)^2}{(1 - A_x)^2}.$$

See Exam 2 review 116 i). Then for payment  $X = 150000$ ,  $V[L_x(X)] = X^2V[L_x(1)]$ . Want  $V[L_x(X)]$ . Here  $L_x(X) = {}_0L_x$ . (May 2007 SOA exam problem 4.)

5) Get the illustrative life table. Assume mortality follows the illustrative life table and  $i = 0.06$ . Find the premium for a fully discrete whole life insurance of 1000 on  $(40)$ .

Hint: see MLC 27 and use Exam 2 review 115 i) to get the premium for 1 unit. Then multiply this formula by 1000. Note that the table gives  $1000A_x$ .