Math 402 HW 7 Spring 2023. Due Friday, March 24. Quiz 7 Wed. March 22. Exam 2 Wednesday March 29. Cumulative Final: Friday, May 12, 10:15-12:15.

1) An auto insurance company divides policyholders into 3 states: 1 for no accidents as a policyholder, 2 for exactly one accident as a policyholder and 3 for two or more accidents as a policyholder. Suppose the transition matrix

$$\boldsymbol{P} = \begin{bmatrix} 0.9 & 0.09 & 0.01 \\ 0 & 0.8 & 0.2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Suppose all policyholders are initially in State 1. Find $\boldsymbol{\pi}_2$, the state vector at time 2 years.

2) Suppose

$$\boldsymbol{P}^{(1)} = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{P}^{(2)} = \begin{bmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{bmatrix}.$$

If the process begins in State 1, what is the probability that the process will be in State 2 after 2 steps?

Hint: see Exam 2 review 137) and 138).

3) For a nonhomogeneous Markov chain with two states, Intact and Failed, the following matrix shows the probability of movement between states, where $t = 1, 2, 3, ...$

$$\boldsymbol{Q}_t = \begin{bmatrix} 0.8^{0.5t} & 1 - 0.8^{0.5t} \\ 0 & 1 \end{bmatrix}$$

Calculate the minimum number of time periods so that the expected percentage of entities in state Intact is less than 10%.

Hints: Let $\boldsymbol{\pi}_n = (p_n, q_n) = \boldsymbol{\pi}_0 \boldsymbol{Q}_1 \boldsymbol{Q}_2 \cdots \boldsymbol{Q}_n$. Want the smallest n such that $p_n < 0.1$. CAS Spring 2009 Exam 3L no. 6.

The following text, along with earlier editions, has a good chapter on Markov chains (ch. 4). The library has several copies including electronic access.

Ross, Sheldon, M. (2007, 2014), *Introduction to Probability Models*, 7th and 11th ed., Academic Press, San Diego, CA.

Google "Daniel Markov Chains" to find some study notes on Markov chains.

https://www.soa.org/files/edu/m-24-05.pdf

Problems 38, 54, 89, 151, 152, 179, 180, 181, 217, 218, 250, 283 from the SOA MLC practice exam may cover Markov Chains.