Math 402 HW 8 Spring 2023. Due Friday, April 7.

1) If the survival model is exponential with failure rate λ and the force of interest is δ, calculate ${}_t\overline{V}(\overline{A}_x)$.

Hint: See Exam 3 review 149) and use the fact that $\overline{a}_{x+t} = \overline{a}_x$ does not depend on t for the exponential distribution.

2) Suppose $\overline{A}_x = 0.4$, ${}^2\overline{A}_x = 0.5$, and $\overline{A}_{x+20} = 0.6$. Find the terminal reserve ${}_{20}\overline{V}(\overline{A}_x)$.

Hint: $\overline{a}_x = \dfrac{1-\overline{A}_x}{\delta}$, and you do not need δ. Also, ${}_t\overline{V}(\overline{A}_x) = 1 - \dfrac{\overline{a}_{x+t}}{\overline{a}_x}$. Or use better formulas like the insurance ratio formula. Exam 3 review 149) above 150) gives some of these formulas.

3) Calculate $\overline{a}_{x+t}$ given ${}_t\overline{V}(\overline{A}_x) = 0.1$, $\overline{P}(\overline{A}_x) = 0.0105$ and $\delta = 0.03$.

Hint: $\overline{A}_{x+t} = 1 - \delta\overline{a}_{x+t}$. Use the formula for ${}_t\overline{V}(\overline{A}_x)$ and solve for $\overline{a}_{x+t}$.

4) For discrete whole life insurance on (40), mortality follows the illustrative life table and $i = 0.06$. Calculate ${}_{17}V_{40}$ using

a) the annuity ratio formula

b) the insurance ratio formula.

Hint: Use Exam 3 review 149)

5) Suppose that for a fully continuous whole life insurance on (40), $\mu_{40}(t) = \dfrac{1}{60-t}$ and $\delta = 0.05$. Find the 20th terminal reserve ${}_{20}\overline{V}(\overline{A}_{40})$.

Hint: Hence for $x = 40$, $T_{x+t} \sim U(60-t)$, and $\overline{A}_{x+t} = E(e^{-\delta T_{x+t}}) = \dfrac{1}{60-t}\displaystyle\int_0^{60-t} e^{-\delta u}du.$

So ${}_t\overline{V}(\overline{A}_x) = \dfrac{\overline{A}_{x+t} - \overline{A}_x}{1-\overline{A}_x}$. Find $\overline{A}_{60} = \overline{A}_{40+20}$ and $\overline{A}_{40} = \overline{A}_{40+0}$.

6) Consider a fully continuous whole life model with payment benefit $b_{t+s} = Je^{\theta(t+s)}$ for $s, t \geq 0$, benefit payment rate is $\overline{P}(t+s) = \pi_0 e^{\gamma(t+s)}$ for $s, t \geq 0$, $\mu_{x+t} \equiv \mu$ for $t > 0$. Suppose $t = 2$, $T_x > 2$, J = 1000, $\mu = 0.02$, $\theta = \delta = 0.04$, and $\gamma = 0$.

a) Find π_0.

b) Find ${}_2\overline{V}$.

Hint: see 157).

7) For a special fully continuous whole life insurance, the benefit is $1000e^{0.02t}$ if death occurs at time t. The annual premium rate is $e^{0.03t}\pi_0$ at time t where π_0 is the premium at time 0, $\mu = 0.01$, and $\delta = 0.06$.

a) Find π_0.

b) Find the terminal reserve at the end of year 5.

Hint: see 157) and 158).

8) Suppose ${}_1V = 1000$, ${}_2V = 1052.63$ and $P = 200$.

a) Approximate ${}_{0.5}V$.

b) Approximate ${}_{1.5}V$. (Hint: see 155).