Math 402 HW 9 Spring 2023. Due Friday, April 14.

| k | $q_k^{(1)}$ | $q_k^{(2)}$ | $q_k^{(3)}$ | ${}_kp_0^{(\tau)}$ | $v^{k+1}$ |
|---|---|---|---|---|---|
| 0 | 0.01 | 0.03 | 0.00 | 1 | 0.943 |
| 1 | 0.01 | 0.04 | 0.00 | 0.96 | 0.89 |
| 2 | 0.01 | 0.05 | 0.00 | 0.912 | 0.84 |
| 3 | 0.01 | 0.06 | 0.00 | 0.857 | 0.792 |
| 4 | 0.01 | 0.00 | 0.99 | 0.797 | 0.747 |

1) A five year bond, issued at time 0, faces the three decrements (1) default, (2) call (prepayment), and (3) maturity. A guarantor has contracted to pay 1000 at the end of the year of default if default occurs, and nothing otherwise. The table above was computed using an annual interest rate of 6%. Find the APV of the guarantor's contract. (See example in class and exam 3 review 160) and 161).)

2) Let $Z$ be the present value random variable for a whole life insurance of $b$ payable at the moment of death of (x). Assume that $\delta = 0.04$, $\mu_{x+t} = 0.02$, $t \geq 0$, and the single benefit premium for this insurance is equal to Var($Z$). Calculate $b$.

Hint: MLC 141. $Z = b\overline{Z}_x$ and $Var(\overline{Z}_x) = E(\overline{Z}_x^2) - [E(\overline{Z}_x)]^2 = {}^2\overline{A}_x - (\overline{A}_x)^2$. The single benefit premium = APV = $E(Z) = Var(Z) = b\overline{A}_x = b^2 Var(\overline{Z}_x)$.

3) (Like MLC 5 example done in class, but using more realistic $\mu$'s). A whole life policy provides that upon an accidental death as a passenger on an airplane (decrement 1), a benefit of 1,000,000 will be paid. If death occurs from other accidental causes (decrement 2), a benefit of 500,000 will be paid. If death occurs from a cause other than an accident (decrement 3), a benefit of 250,000 will be paid. Death benefits are payable at moment of death. Assume $\mu^{(1)} = \frac{1}{2,000,000}$, $\mu^{(2)} = \frac{1}{10,000}$, $\mu^{(3)} = \frac{1}{40}$, and $\delta = 0.06$. Find the single benefit premium.

4) For a special fully continuous last survivor whole life insurance of 1 on (x) and (y), the premium is payable until first death, and the independent random variables $T_x^*, T_y^*$, and $Z$ are components of the common shock model. Assume $T_x^* \sim EXP(0.04), T_y^* \sim EXP(0.06)$, and $Z \sim EXP(0.02)$. If $\delta = 0.04$, find the annual benefit premium.

Hint: $\lambda = 0.02$, use 165) and 166) from the exam 3 review. MLC 26) and 49) are similar, but do not use the common shock model.

5) For a fully discrete whole life insurance of 1000 on (60), you are given

i)

| Expense Type | First Year | Renewal Years |
|---|---|---|
| % of Premium | 20% | 6% |
| Per Policy | 8 | 2 |

ii) the gross premium is 41.20, iii) i = 0.04,

iv) ${}_0L$ is the present value of the loss random variable at issue.

Calculate ${}_0L$ if the insured dies in the 3rd policy year.

Hint: This is like the MLC 198 example done in class.

6) One asset shares problem gives all of the variables except one, usually $i$, and has

you solve for the remaining variable. The following example is MLC 236, and the solution is below.

For a fully discrete insurance of 1000 on (x), you are given: i) ${}_4AS = 396.63$ is the asset share at the end of year 4.
ii) ${}_5AS = 694.5$ is the asset share at the end of year 5.
iii) G = 281.77 is the gross premium.
iv) ${}_5CV = 572.12$ is the cash value at the end of year 5.
v) $c_4 = 0.05$ is the fraction of the gross premium paid at time 4 for expenses.
vi) $e_4 = 7.0$ is the amount of per policy expenses paid at time 4.
vii) $q_{x+4}^{(1)} = 0.09$ is the probability of decrement by death.
viii) $q_{x+4}^{(2)} = 0.26$ is the probability of decrement by withdrawal.

Calculate $i$.

Note: MLC 242 (also see 235 and 244) is similar but gave $i$ and everything on the right hand side of the formula.

likely used for HW9

**Question #236**
**Answer: A**

$$ {}_5AS = \frac{\left({}_4AS + G(1-c_4) - e_4\right)(1+i) - 1000q_{x+4}^{(1)} - {}_5CV \times q_{x+4}^{(2)}}{1 - q_{x+4}^{(1)} - q_{x+4}^{(2)}} $$

$$ = \frac{\left(396.63 + 281.77(1-0.05) - 7\right)(1+i) - 90 - 572.12 \times 0.26}{1 - 0.09 - 0.26} $$

$$ = \frac{(657.31)(1+i) - 90 - 148.75}{0.65} $$

$$ = 694.50 $$

$$ (657.31)(1+i) = 90 + 148.75 + (0.65)(694.50) $$

$$ 1 + i = \frac{690.18}{657.31} = 1.05 $$

$$ i = 0.05 $$

**Question #237 - Removed**

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