$$\left(\frac{n}{d}\right)' = \frac{dn' - d'n}{d^2}$$

Math 402 Quiz 1 Spring 2019 Name____________________

YOU ARE BEING GRADED FOR WORK

1) Suppose ${}_tp_x = S_x(t) = \dfrac{1+x}{1+x+t}$ where $t > 0$ and $x > 0$ is fixed. Find the following quantities for $t > 0$.

a) ${}_tq_x = 1 - {}_tp_x = \boxed{1 - \frac{1+x}{1+x+t} = \frac{t}{1+x+t}}$

b) $f_x(t) = \frac{d}{dt}\,{}_tq_x = \frac{(1+x+t)1 - 1(t)}{(1+x+t)^2} = \boxed{\frac{1+x}{(1+x+t)^2}}$

c) μ_{x+t} $= \frac{f_x(t)}{S_x(t)} = \frac{1+x}{(1+x+t)^2}\,\frac{1+x+t}{1+x} = \boxed{\frac{1}{1+x+t}}$

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2) Suppose $T_x \sim EXP(\mu)$. Find ${}^2\overline{A}_x = E[(\overline{Z}_x)^2] = E[(v^T)^2] = E(e^{-2\delta T}) = \int_0^\infty v^{2t} f_T(t)\,dt = \int_0^\infty e^{-2\delta t}\mu e^{-\mu t}\,dt.$

$= \mu\int_0^\infty e^{-t(\mu+2\delta)}\,dt$ $= \frac{\mu}{\mu+2\delta}\underbrace{\int_0^\infty (\mu+2\delta)\,e^{-t(\mu+2\delta)}\,dt}_{1 = \int_0^\infty EXP(\mu+2\delta)\,pdt}$

$= \boxed{\frac{\mu}{\mu+2\delta}}$

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$b_t v_t = e e = e, \quad b_t = e^{\theta t}, \theta = .03$

3) For a special whole life insurance on (x), payable at the moment of death, $\mu_{x+t} = 0.02, t > 0$, $\delta = 0.08$, the death benefit at time t is $b_t = e^{0.03t}, t > 0$, and Z is the present value random variable for this insurance at issue.

a) Find $E(Z)$. $= \dfrac{\mu}{\mu+\delta-\theta} = \dfrac{.02}{.02+.08-.03} = \boxed{\dfrac{2}{7} = 0.2857}$

hard way

$= E(b_T v^T) = \int_0^\infty e^{-.05t} .02 e^{-.02t} dt = \int_0^\infty e^{.03t} e^{-.08t} .02 e^{-.02t} dt = .02\int_0^\infty e^{-.07t} dt = \dfrac{.02}{.07}$

b) Find $V(Z)$. $E(Z^2) = \dfrac{\mu}{\mu+2\delta-2\theta} = \dfrac{.02}{.02+2(.08)-2(.03)} = \dfrac{2}{12} = \dfrac{1}{6} = .1667$

$V(Z) = E(Z^2) - [E(Z)]^2 = \frac{1}{6} - \left(\frac{2}{7}\right)^2 = \boxed{0.08503}$

hard way

$E(Z^2) = E[(b_T v^T)^2] = \int_0^\infty \underbrace{e^{-.1t}}_{e^{.06t}\, e^{-.16t}} .02 e^{-.02t} = .02\int_0^\infty e^{-.12t} dt = \dfrac{.02}{.12}$

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4) Consider an n year pure endowment insurance where the force of interest is δ. Find ${}_nE_x = v^n P(T_x > n) = v^n S_x(n) = v^n \ {}_np_x = A_{\substack{1\\x:\overline{n}|}}$ if $S_x(t) = {}_tp_x = \dfrac{\exp[\theta(1-e^{\alpha(x+t)})]}{\exp[\theta(1-e^{\alpha x})]}$ where $t, x, \theta, \alpha > 0$.

Gompertz

$= e^{-\delta n} S_x(n) =$

$$\boxed{e^{-\delta n} \frac{\exp[\theta(1-e^{\alpha(x+n)})]}{\exp[\theta(1-e^{\alpha x})]}}$$

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$= e^{-\delta n} \exp[\theta(1-e^{\alpha(x+n)}) - \theta(1-e^{\alpha x})] = e^{-\delta n} \exp[\theta(e^{\alpha x} - e^{\alpha(x+n)})]$