

need $\mu + \delta > \max(\theta, \gamma)$

old Q10

Math 402 Quiz 8 Spring 2015

Name _____

$J = 10000, \theta = 0.005, \gamma = .03$

1) For a special fully continuous whole life insurance, the benefit is $10000e^{0.005t}$ if death occurs at time t . The annual premium rate is $e^{0.03t}\pi_0$ at time t where π_0 is the premium at time 0. $\mu = 0.02$, and $\delta = 0.04$.

a) Find π_0 .

$$= \frac{J \mu (\mu + \delta - \gamma)}{(\mu + \delta - \theta)} = \frac{10000(0.02)(.02 + .04 - .03)}{(.02 + .04 - .005)}$$

$$= \frac{6}{.055} = \boxed{109.0909}$$

b) Find the benefit reserve at the end of year 5.

$${}_5V = \frac{10000(0.02) e^{.005(5)}}{.02 + .04 - .005} - \frac{109.0909 e^{.03(5)}}{.02 + .04 - .03}$$

$$= \frac{200 e^{.025}}{.055} - \frac{109.0909 e^{.15}}{.03} =$$

$$3728.4186 - 4224.8514 = \boxed{-496.4328}$$

2) Suppose a population consists of 30% smokers (S) with $\mu_S = 0.07$ and 70% non-smokers (NS) with $\mu_{NS} = 0.03$. Find \bar{A}_x for this population if $\delta = 0.04$.

Find \bar{a}_{xy} instead

$$.3 \bar{A}_x(S) + .7 \bar{A}_x(NS) = \frac{.41}{\mu + \delta} \cdot .3 + \frac{.47}{\mu + \delta} \cdot .7$$

$$= \frac{.07}{.07 + .04} \cdot .3 + \frac{.03}{.03 + .04} \cdot .7 = \frac{7}{11} \cdot .3 + \frac{3}{7} \cdot .7$$

$$= \boxed{0.4909}$$

010010

$$P = \begin{bmatrix} & H & S & D \\ H & 0.7 & 0.2 & 0.1 \\ S & 0.3 & 0.5 & 0.2 \\ D & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

1) Suppose the above transition matrix is for a homogeneous Markov chain with states healthy (H), sick (S), and dead (D). Assume each insured is initially healthy and that $v = 0.9$.

a) Assume the insurance pays 3000 (on average) at the beginning of the year if the person is sick at the beginning of the year. Find the APV of the insurance for the next 2 years. The time diagram below may be useful.

	0	3000	3000

	0	1	2
P(Sick)		.2	.24

$$\pi_0 = [1 \quad 0 \quad 0]$$

$$\pi_1 = [0.7 \quad .2 \quad .1]$$

$$\pi_2 = [0.55 \quad .24 \quad .21]$$

$$(.7(.7) + .2(.3)) \quad .7(.2) + .2(.5) \quad .7(.1) + .2(.2) + .1(1)$$

$$APV = \sum c_i v^i p_i = 3000(0.9)^0 \cdot 0.2 + 3000(0.9)^1 \cdot 0.24 = 3000(0.3744) = 1123.20$$

b) Suppose a gross premium of 800 is paid at the beginning of each year the person is healthy or sick. Find the APV of the premiums over the first two years (three possible premiums). The time diagram below may be useful.

	800	800	800

	0	1	2
P(Healthy)		.9	.79
		.1	.11
	1+0	.2+2	.55+.24

$$APV = \sum c_i v^i p_i =$$

$$800 \left[\underbrace{(0.9)^0}_1 + (0.9)^1 \cdot 0.9 + (0.9)^2 \cdot 0.79 \right]$$

P(Healthy)

$$= 800 (2.4499) = 1959.92$$

1959.2