old QUIZZ sometimes 401

Math 402 Quiz 2 Spring 2015 Name____________________

e 1) Assume the UDD assumption holds so ${}_tp_x \approx 1 - t(q_x)$. For independent lives (x) and (y), suppose $q_x = 0.04$ and $q_y = 0.08$.

a) Use the UDD approximation for ${}_{0.5}p_x$ and ${}_{0.5}p_y$. 401fd18

$${}_{.5}p_x \approx 1-(.5)(.04) = \boxed{0.98}$$

$${}_{.5}p_y \approx 1-(.5)(.08) = \boxed{0.96}$$

b) Then approximate ${}_{0.5}p_{xy}$ and ${}_{0.5}q_{xy}$.

$${}_{.5}p_{xy} \approx ({}_{.5}p_x)({}_{.5}p_y) = .98(.96) = \boxed{.9408}$$

$${}_{.5}q_{xy} = 1 - {}_{.5}p_{xy} \approx \boxed{0.0592}$$

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e 2) Suppose $T_x \sim EXP(0.02)$, $T_y \sim EXP(0.03)$ and T_x and T_y are independent.

a) Find $\overset{\circ}{e}_{xy}$.

$$T_{xy} \sim EXP(.02+.03) = EXP(0.05)$$

$$\text{so } \overset{\circ}{e}_{xy} = E(T_{xy}) = \frac{1}{.05} = \boxed{20}$$

b) Find $\overset{\circ}{e}_{\overline{xy}} = \overset{\circ}{e}_x + \overset{\circ}{e}_y - \overset{\circ}{e}_{xy}$.

see 401F18

$$= \frac{1}{.02} + \frac{1}{.03} - 20$$

$$= 50 + 33.3333 - 20 = \boxed{63.3333}$$

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3) Suppose ${}_tp_x = {}_tp_y = 0.01(10-t)^2$ for $0 < t < 10$ and that ${}_tp_x = {}_tp_y = 0$ for $t \geq 10$. Assume $T_x \perp\!\!\!\perp T_y$.

e a) Find ${}_tp_{xy}$.

$= {}_tp_x \, {}_tp_y = [.01\,(10-t)^2]^2$

$= \boxed{.0001\,(10-t)^4}$

b) Find $\overset{o}{e}_{xy} = \int_0^{10} {}_tp_{xy}dt$. $= \int_0^{10} .0001\,(10-t)^4\,dt$

$(u = 10-t \quad du = -dt, \quad t=0 \to u=10, \; t=10 \to u=0)$

$= -\int_{10}^{0} .0001\,u^4\,du = \int_0^{10} .0001\,u^4\,du =$

$.0001 \left.\frac{u^5}{5}\right|_0^{10} = \frac{10^{-4}\,10^5}{5} = \frac{10}{5} = \boxed{2}$

or $= .0001\left(\underbrace{\left.-\frac{(10-t)^5}{5}\right|_0^{10}}_{\text{antiderivative}}\right) = \frac{10^{-4}\,10^5}{5} = \frac{10}{5}$

$\frac{\quad}{15}$

4) Suppose $(80) \perp\!\!\!\perp (85)$. What is the correct formula for the probability that the last death occurs after 5 and before 10 years from now? (Circle the letter corresponding to the best answer.)

a) $P(5 < T_{xy} \leq 10) = P(T_{xy} > 5) - P(T_{xy} > 10) = {}_5p_{80:85} - {}_{10}p_{80:85}$
b) $P(5 < T_{\overline{xy}} \leq 10) = P(T_{\overline{xy}} > 5) - P(T_{\overline{xy}} > 10) = {}_5p_{\overline{80:85}} - {}_{10}p_{\overline{80:85}}$
c) $P(5 < T_{xy} \leq 10) = P(T_{xy} > 5) - P(T_{xy} > 10) = {}_5q_{80:85} - {}_{10}q_{80:85}$
d) $P(5 < T_{\overline{xy}} \leq 10) = P(T_{\overline{xy}} > 5) - P(T_{\overline{xy}} > 10) = {}_5q_{\overline{80:85}} - {}_{10}q_{\overline{80:85}}$

$\frac{\quad}{10}$

e 5) If $T_{80} \perp\!\!\!\perp T_{85}$, ${}_{10}p_{80} = 0.05$ and ${}_{10}p_{85} = 0.04$, what is ${}_{10}p_{80:85}$?

$.05(.04) = \boxed{.0020}$

$\frac{\quad}{25}$