old $T_{xy} \sim EXP(\mu_x + \mu_y)$ now 401

Math 402 Quiz 3 Spring 2015 Name____________________

review for 401 Q11 401 final8

1) Suppose $T_x \sim EXP(0.05) \perp\!\!\!\perp T_y \sim EXP(0.03)$ and $\delta = 0.04$.
a) Find $\overline{A}_{xy}$.

$$= \frac{\mu_x + \mu_y}{\delta + \mu_x + \mu_y} = \frac{.08}{.04 + .08} = \frac{2}{3} = \boxed{0.6667}$$

b) Find ${}^2\overline{A}_{xy}$.
$$= \frac{\mu_x + \mu_y}{2\delta + \mu_x + \mu_y} = \frac{0.08}{0.08 + 0.08} = \boxed{0.5}$$

c) Find $\overline{a}_{xy}$.
$$= \frac{1}{\delta + \mu_x + \mu_y} = \frac{1}{0.12} = \frac{100}{12} = \boxed{8.3333}$$

d) Find $V[\overline{Y}_{xy}]$.
$$= \frac{{}^2\bar{A}_{xy} - (\bar{A}_{xy})^2}{\delta^2} =$$

$$\frac{0.5 - \frac{4}{9}}{(0.04)^2} = \boxed{34.7222}$$

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2) You are given $T_x \sim EXP(\mu) \perp\!\!\!\perp T_y \sim EXP(\lambda)$. Determine the probability ${}_nq^{\,1}_{xy}$ that (x) dies before (y) and within n years.

$$= \int_0^n {}_tp_x \, {}_tp_y \, \mu_{x+t} \, dt = \int_0^n e^{-t\mu} e^{-t\lambda} \mu \, dt = \mu \int_0^n e^{-t(\mu+\lambda)} dt$$

$$= \mu \frac{e^{-t(\mu+\lambda)}}{-(\mu+\lambda)} \Big|_0^n = \frac{-\mu}{\mu+\lambda} \left[e^{-n(\mu+\lambda)} - 1 \right] = ?$$

$$\underline{\text{or}} = \frac{\mu}{\mu+\lambda} \int_0^n \underbrace{(\mu+\lambda) e^{-t(\mu+\lambda)}}_{EXP(\mu+\lambda)\ pdf} dt = \frac{\mu}{\mu+\lambda} F_{\mu+\lambda}(n) = \boxed{\frac{\mu}{\mu+\lambda} \left(1 - e^{-n(\mu+\lambda)}\right)}$$

3) Suppose $T_x \sim U(0, 20) \perp\!\!\!\perp T_y \sim U(0, 40)$. Let $a = \min(\omega_x - x, \omega_y - y)$ and $b = \max(\omega_x - x, \omega_y - y)$.

a) Find $\overset{\circ}{e}_{xy} = \dfrac{a}{2} - \dfrac{a^2}{6b}$.

$$= \frac{20}{2} - \frac{(20)^2}{6(40)} = 10 - 1.6667$$

$$= \boxed{8.3333}$$

b) Find $\overset{\circ}{e}_{\overline{xy}} = \dfrac{b}{2} + \dfrac{a^2}{6b}$.

$$= \frac{40}{2} + \frac{(20)^2}{6(40)} = 20 + 1.6667$$

$$= \boxed{21.6667}$$