Math 402 Quiz 4 Spring 2015 Name____________________

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(\tau)}$ ($q_x^{(1)}+q_x^{(2)}$)	$p_x^{(\tau)}$ ($1-q_x^{(\tau)}$)	$l_x^{(\tau)}$ ($p_{x-1}^{(\tau)}\, l_{x-1}^{(\tau)}$)	$d_x^{(1)}$ ($l_x^{(\tau)}\, q_x^{(1)}$)	$d_x^{(2)}$ ($l_x^{(\tau)}\, q_x^{(2)}$)
45	0.014	0.1	0.114	0.886	1000	14	100
46	0.015	0.1	0.115	0.885	886	13.29	88.6
47	0.016	0.1	0.116	0.884	784.11	12.5458	78.411
48	0.017	0.1	0.117	0.883	693.1532	11.7836	69.315
49	0.018	0.1	0.118	0.882	612.0543	11.0170	61.205
50	0.019	0.1	0.119	0.881	539.8319	10.2568	53.983

1) a) Complete the above table. Note that the initial group size is 1000.

b) Find ${}_3p_{46}^{(\tau)}$.

$$\frac{l_{x+n}^{(\tau)}}{l_x^{(\tau)}} = \frac{l_{49}^{(\tau)}}{l_{46}^{(\tau)}} = \frac{612.0543}{886} = \boxed{0.6908}$$

c) Find ${}_{2|}q_{46}^{(\tau)}$.

$$ {}_{k|}q_x^{(\tau)} = \frac{\sum_{j=1}^{m} d_{x+k}^{(j)}}{l_x^{(\tau)}} = \frac{d_{48}^{(1)} + d_{48}^{(2)}}{l_{46}^{(\tau)}} = \frac{11.7876+69.315}{886} = \boxed{0.0915}$$

(from a)

$$\text{or} = {}_kp_x^{(\tau)} - {}_{k+1}p_x^{(\tau)} = \frac{l_{48}^{(\tau)} - l_{49}^{(\tau)}}{l_{46}^{(\tau)}} = \frac{693.1532 - 612.0543}{886}$$

d) Find ${}_{2|}q_{46}^{(1)}$.

$$ {}_{k|}q_x^{(j)} = \frac{d_{x+k}^{(j)}}{l_x^{(\tau)}} \quad \text{so} \quad {}_{2|}q_{46}^{(1)} = \frac{d_{48}^{(1)}}{l_{46}^{(\tau)}} = $$

see 75 xiv)

$$\frac{11.7836}{886} = \boxed{0.01330}$$

866

50

$$\text{or} = {}_2p_{46}^{(\tau)}\, q_{48}^{(1)} = \frac{l_{48}^{(\tau)}}{l_{46}^{(\tau)}}(0.017) = \frac{693.1532}{886}\, 0.017 =$$

2) A multiple decrement model with two causes of decrement has $\mu_{x+t}^{(1)} = 0.02$ and $\mu_{x+t}^{(2)} = 0.03$ where $t > 0$.

a) Find $\mu_{x+t}^{(\tau)}$. $= \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)} = \boxed{0.05}$

μ of $EXP(0.05)$ RV

b) Find ${}_tp_x^{(\tau)}$. $= \boxed{e^{-0.05t}}$, $t>0$ = $EXP(0.05)$ survival function

or $= \exp\left[-\int_0^t .05\,ds\right] = \exp\left[-.05\, s \Big|_0^t\right] = e^{-0.05t}$

or $= {}_tp_x'^{(1)} \; {}_tp_x'^{(2)} = e^{-.02t}\, e^{-.03t} = e^{-.05t}$

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3) For the **illustrative service table**, the four decrements are (d) for death, (w) for withdrawal, (i) for disability, and (r) for retirement. Use this table to find the following quantities.

handout

a) Find ${}_4p_{40}^{(\tau)}$. ${}_np_x^{(T)} = \dfrac{l_{x+n}^{(T)}}{l_x^{(T)}}$ so want $\dfrac{l_{44}^{(T)}}{l_{40}^{(T)}} = \dfrac{33659}{36943}$

$= \boxed{0.9111}$

b) Find ${}_4d_{40}^{(d)}$. ${}_nd_x^{(j)} = \sum_{t=0}^{n-1} d_{x+t}^{(j)} = d_x^{(j)} + d_{x+1}^{(j)} + \dots + d_{x+n-1}^{(j)}$

so ${}_4d_{40}^{(d)} = d_{40}^{(d)} + d_{41}^{(d)} + d_{42}^{(d)} + d_{43}^{(d)}$

$= 78 + 83 + 91 + 96 = \boxed{348}$

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