old Q5 also see back of HW5

Math 402 Quiz 7 Spring 2019 Name________________

k	$q_k^{(1)}$	$q_k^{(2)}$	$q_k^{(3)}$	${}_kp_0^{(\tau)}$	v^{k+1}
0	0.02	0.03	0.00	1	0.943
1	0.02	0.04	0.00	0.95	0.89
2	0.02	0.05	0.00	0.893	0.84
3	0.02	0.06	0.00	0.83	0.792
4	0.02	0.00	0.98	0.764	0.747

may or may not be on Q5

1) A five year bond, issued at time 0, faces the three decrements (1) default, (2) call (prepayment), and (3) maturity. A guarantor has contracted to pay 1000 at the end of the year of default if default occurs, and nothing otherwise. The table above was computed using an annual interest rate of 6%. Find the $APV = 1000\sum_{k=0}^{4} v^{k+1}\ {}_kp_0^{(\tau)} q_k^{(1)}$ of the guarantor's contract.

$APV = \bar{A}^{(1)} = \sum_{k=0}^{4} v^{k+1}\ {}_kp_0^{(\tau)}\ q_{0+k}^{(1)}$

$= 1000[.943(1).02 + .89(.95).02 + .84(.893).02 + .792(.83).02 + .747(.764).02]$

$= \boxed{75.3338}$

2) For the common shock model with $\lambda = 0.01$, assume that $T_x^* \sim EXP(0.03)$, and $T_Y^* \sim EXP(0.04)$. Consider a last survivor whole life insurance of 1000 on (x) and (y). Find the APV of this insurance if $\delta = 0.02$ and the death benefit is payable immediately after the 2nd death. Hint: want $1000\bar{A}_{\overline{xy}}$.

$T_x \sim EXP(\mu_x^* + \lambda) \sim EXP(.04)$

$T_y \sim EXP(\mu_y^* + \lambda) \sim EXP(.05)$

$T_{xy} \sim EXP(\mu_x^* + \mu_y^* + \lambda) \sim EXP(.08)$

$\bar{A}_w = \frac{\mu}{\mu+\delta}$

$= 1000(\bar{A}_x + \bar{A}_y - \bar{A}_{xy})$

$= 1000\left(\frac{.04}{.04+.02} + \frac{.05}{.05+.02} - \frac{.08}{.08+.02}\right)$

$= 1000\left(\frac{4}{6} + \frac{5}{7} - \frac{8}{10}\right) = \boxed{580.9524}$

$= 1000\left(\frac{2}{3} + \frac{5}{7} - \frac{4}{5}\right)$

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023

012 Q5

3) Suppose that for a triple decrement model and $t > 0$, $\mu_x^{(1)}(t) = 0.2$, $\mu_x^{(2)}(t) = 0.5$, and $\mu_x^{(3)}(t) = 0.5$. Find $q_x^{(3)}$.

$$\mu_x^{(\tau)} = .2 + .5 + .5 = \underbrace{1.2}_{EXP(1.2)} = \sum_{j=1}^{3} \mu^{(j)}(t)$$

$${}_1q_x^{(3)} = q_x^{(3)} \text{ has } t=1, \text{ so } q_x^{(3)} = \int_0^1 {}_sp_x^{(\tau)} \mu_{x+s}^{(3)} ds$$

$$= \int_0^1 e^{-1.2s} 0.5 \, ds = 0.5 \left. \frac{e^{-1.2s}}{-1.2} \right|_0^1 =$$

$$\frac{0.5}{-1.2}(e^{-1.2} - 1) = \frac{0.5}{1.2}(1 - e^{-1.2}) = \boxed{0.2912}$$

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4) In a triple decrement model, $\mu_x^{(1)}(t) = 1/(50-t)$ for $0 < t < 50$, $\mu_x^{(2)}(t) = 0.01$ for $t > 0$, and $\mu_x^{(3)}(t) = 0.002t$ for $t > 0$. Given that someone fails at time $t = 30$, find the probability that the person failed due to decrement 3.

$$\mu_x^{(\tau)}(30) = \mu_x^{(1)}(30) + \mu_x^{(2)}(30) + \mu_x^{(3)}(30)$$

$$= \underbrace{\frac{1}{50-30}}_{.05} + .01 + \underbrace{.002(30)}_{.06} = 0.12$$

$$\text{So } \frac{\mu_x^{(3)}(30)}{\mu_x^{(\tau)}(30)} = \frac{0.06}{0.12} = 0.5$$

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