$y_j = x_j$ $s_j = d_j$

$S_n(t) = \hat{S}(t)$

Math 403 Quiz 10 Fall 2017
YOU ARE BEING GRADED FOR WORK

Name____________________

$= \prod_{i=1}^{j}\left(1-\frac{s_i}{r_i}\right) = S_n(y_{j-1})\left(1-\frac{s_j}{r_j}\right)$

j	y_j	s_j (d_j)	r_j	$S_n(y_j)$	
1	1	3	15	$\frac{12}{15}$ =	$\boxed{0.8}$
2	2	24	80	$.8\left(\frac{56}{80}\right)$ =	$\boxed{0.56}$
3	3	5	25	$.56\left(\frac{20}{25}\right)$ =	$\boxed{0.448}$
4	4	6	60	$.448\left(\frac{54}{60}\right)$ =	$\boxed{0.4032}$
5	5	3	10	$.4032\left(\frac{7}{10}\right)$ =	$\boxed{0.2822}$

~16

1) a) Fill in the above table, where $S_n(y_j)$ is the Kaplan Meier product limit estimator.
b) Find $\hat{H}(5)$.

$= \sum_{i=1}^{5}\frac{s_i}{r_i} = \frac{3}{15}+\frac{24}{80}+\frac{5}{25}+\frac{6}{60}+\frac{3}{10} = \boxed{1.1}$

12

c) Use b) to find $\hat{S}(5)$. $e^{-\hat{H}(5)} = e^{-1.1} = \boxed{0.3329}$

12

d) Find $S_n(5)$. $= \boxed{0.2822}$

12

e) Determine the Greenwood approximation for the variance of the Kaplan Meier product limit estimator $S_n(4)$.

$\hat{V}(S_n(4)) = [S_n(4)]^2 \sum_{i=1}^{4}\frac{s_i}{r_i(r_i-s_i)} =$

$(.4032)^2\left[\frac{3}{15(15-3)}+\frac{24}{80(80-24)}+\frac{5}{25(25-5)}+\frac{6}{60(60-6)}\right]$

$= (.4032)^2(0.03388) = \boxed{0.005507}$

12

64

ignore 403 Fall 17

2) Suppose the observed data is 8, 10, 10, 11, 11, 11, 12, 13, 13, 15. Suppose a uniform kernel is used. Find $\hat{f}(12)$ if $b = 2$.

$$= \frac{\# x_i \in [x-b, x+b]}{2(n)b} = \frac{\# x_i \in [10, 14]}{2(10)2} = \frac{8}{40} = \frac{2}{10} = \boxed{0.2} = \frac{1}{5}$$

12

j	y_j	s_j	r_j
1	1	5	40
2	2	4	33
3	4	3	30
4	6	4	22
5	7	2	12

"$s_j = d_j$"

$S_n(4) = \hat{S}_n(4)$

3) Refer to the above table.

a) Find $S_n(5)$. $= S_n(4) = \prod_{j=1}^{3}\left(1 - \frac{s_j}{r_j}\right) = \frac{35}{40}\,\frac{29}{33}\,\frac{27}{30} = \boxed{0.6920}$

b) Find $\hat{V}(S_n(5))$. $= (S_n(5))^2 \sum_{j=1}^{3} \frac{s_j}{r_j(r_j - s_j)} =$

$$(0.692)^2\left[\frac{5}{40(40-5)} + \frac{4}{33(33-4)} + \frac{3}{30(30-3)}\right] = (0.692)^2(0.01145)$$
$$= \boxed{0.005485}$$

c) Find the linear 95% CI for $S(5)$.

$$S_n(5) \pm z_p \sqrt{\hat{V}(S_n(5))} = 0.692 \pm 1.96\sqrt{0.005485} =$$
$$0.692 \pm .1452 = \boxed{[0.5468, 0.8372]}$$

ignore d) Find the log transformed 95% CI for $S(5)$.

$$U = \exp\left(\frac{z_p\sqrt{\hat{V}(S_n(5))}}{S_n(5)\ln(S_n(5))}\right) = \exp\left(\frac{0.1452}{.692\ln(.692)}\right) = e^{-0.5699}$$
$$= .5656,\quad CI = \left([S_n(5)]^{\frac{1}{U}}, [S_n(5)]^{U}\right) =$$
$$\left((.692)^{\frac{1}{.5656}}, (.692)^{.5656}\right) = \boxed{(0.5216, 0.8120)}$$

48