Math 402 Quiz 7 Spring 2019 01029 Name___________________

k	$q_k^{(1)}$	$q_k^{(2)}$	$q_k^{(3)}$	${}_kp_0^{(\tau)}$	v^{k+1}
0	0.02	0.03	0.00	1	0.943
1	0.02	0.04	0.00	0.95	0.89
2	0.02	0.05	0.00	0.893	0.84
3	0.02	0.06	0.00	0.83	0.792
4	0.02	0.00	0.98	0.764	0.747

see Q4d23

1) A five year bond, issued at time 0, faces the three decrements (1) default, (2) call (prepayment), and (3) maturity. A guarantor has contracted to pay 1000 at the end of the year of default if default occurs, and nothing otherwise. The table above was computed using an annual interest rate of 6%. Find the $APV = 1000\sum_{k=0}^{4} v^{k+1}\ {}_kp_0^{(\tau)}\ q_k^{(1)}$ of the guarantor's contract.

$$APV = \bar{A}^{(1)} = \sum_{k=0}^{4} v^{k+1}\ {}_kp_0^{(\tau)}\ q_{0+k}^{(\tau)}$$

$$= 1000[.943(1).02 + .89(.95).02 + .84(.893).02 + .792(.83).02 + .747(.764).02]$$

$$= \boxed{75.3338}$$

2) For the common shock model with $\lambda = 0.01$, assume that $T_x^* \sim EXP(0.03)$, and $T_Y^* \sim EXP(0.04)$. Consider a last survivor whole life insurance of 1000 on (x) and (y). Find the APV of this insurance if $\delta = 0.02$ and the death benefit is payable immediately after the 2nd death. Hint: want $1000\bar{A}_{\overline{xy}}$.

$T_x \sim EXP(\mu_x^* + \lambda) \sim EXP(.04)$

$T_y \sim EXP(\mu_y^* + \lambda) \sim EXP(.05)$

$T_{xy} \sim EXP(\mu_x^* + \mu_y^* + \lambda) \sim EXP(.08)$

$\bar{A}_w = \frac{\mu}{\mu + \delta}$

$$= 1000(\bar{A}_x + \bar{A}_y - \bar{A}_{xy})$$

$$= 1000\left(\frac{.04}{.04+.02} + \frac{.05}{.05+.02} - \frac{.08}{.08+.02}\right)$$

$$= 1000\left(\frac{4}{6} + \frac{5}{7} - \frac{8}{10}\right) = \boxed{580.9524}$$

$$= 1000\left(\frac{2}{3} + \frac{5}{7} - \frac{4}{5}\right)$$

$.1G+9 = 10...$

$.06G+2 = 7.472$ $\qquad v = (1+i)^{-1}$

3) Suppose the insured died in 3rd policy year, and for the insurance APV(gross premiums) $= G(1 + v + v^2)$, and
APV(benefits + expenses) $= 0.1G + 9 + (0.06G + 2)v + (0.06G + 2)v^2 + 1000v^3$.
Find the observed loss for the insured ${}_0L$ if $G = 91.2$ and $i = 0.04$.

${}_0L$ = APV (benefits + expenses) − APV (gross premiums)

$= 18.12 + 7.472(1.04)^{-1} + 7.472(1.04)^{-2} + 1000(1.04)^{-3}$

$- 91.2[1 + (1.04)^{-1} + (1.04)^{-2}]$

$= 921.2093 - 263.2118$

$= \boxed{657.9975}$

4) Suppose $T_x \sim EXP(0.03)$ and $\delta = 0.02$. Find $P(\overline{Z}_x \leq 0.5)$.

$P(\overline{Z}_x \le z) = z^{\mu/\delta}$

$= 0.5^{\frac{.03}{.02}} = 0.5^{1.5} = \boxed{0.3536}$

$T_{xy} \sim EXP(.07 = .04 + .03)$ $\qquad \bar{a}_w = \frac{1}{\mu_w + \delta}$

2) Suppose $T_x \sim EXP(0.04) \perp\!\!\!\perp T_y \sim EXP(0.03)$ and $\delta = 0.01$. Find $\bar{a}_{x|y}$.

$$= \bar{a}_y - \bar{a}_{xy} = \frac{1}{.03+.01} - \frac{1}{.07+.01} = \frac{1}{.04} - \frac{1}{.08} = \frac{1}{.08}$$

$$= \boxed{12.5}$$

5k

3) For a fully discrete insurance of 10000 on (40), you are given:

i) ${}_{15}AS = 1150$ and ${}_{16}AS = 1320$. $\qquad k = 16, \; k-1 = 15, \; x+k-1 = 55$

ii) $i = 0.08$. $\qquad x = 40$

iii) G = 90 is the gross premium.

iv) $c_{15} = 0.05$ is the fraction of the gross premium paid at time 15 for expenses.

v) $e_{15} = 0.0$ is the amount of per policy expenses paid at time 15.

vi) $q_{55}^{(1)} = 0.004$ is the probability of decrement by death.

vii) $q_{55}^{(2)} = 0.05$ is the probability of decrement by withdrawal.

Calculate ${}_{16}CV$, the cash value payable upon withdrawal an the end of year 16.

$${}_kAS = \frac{\left[{}_{k-1}AS + G(1 - c_{k-1}) - e_{k-1}\right](1+i) - b_k \, q^{\overset{1}{(d)}}_{x+k-1} - {}_kCV \, q^{\overset{2}{(w)}}_{x+k-1}}{1 - q^{(d)}_{x+k-1} - q^{(w)}_{x+k-1}}$$

$$1320 = \frac{[1150 + 90(.95)]\,1.08 - 10000(.004) - {}_{16}CV(.05)}{1 - .004 - .05}$$

$$= \left[1334.34 - 40 - ({}_{16}CV).05\right] / \, 0.946$$

$$\text{or } 1248.72 = 1294.35 - .05\,({}_{16}CV)$$

$$\text{So} \quad {}_{16}CV = \frac{1294.34 - 1248.72}{.05} = \frac{45.62}{.05} = \boxed{912.40}$$

(or $.05\,({}_{16}CV) = 1294.34 - 0.946(1320)$

${}_{16}CV = 912.40$)