ii) fully continuous whole life

$${}_t\bar{V}(\bar{A}_x) = \bar{A}_{x+t} - \left[\bar{P}(\bar{A}_x)\right]\bar{a}_{x+t} =$$

$$\bar{a}_{x+t}\left[\bar{P}(\bar{A}_{x+t}) - \bar{P}(\bar{A}_x)\right] = \bar{A}_{x+t}\left(1 - \frac{\bar{P}(\bar{A}_x)}{\bar{P}(\bar{A}_{x+t})}\right)$$

$$= 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

p239–240

21} For whole life and endowment insurance

i) $${}_tV = 1 - \frac{a_{x+t}}{a_x},$$ the annuity ratio formula.

ii) $${}_tV = \frac{A_{x+t} - A_x}{1 - A_x}$$ is the insurance ratio formula

iii) $${}_tV = \frac{P_{x+t} - P_x}{P_{x+t} + d}$$ is the premium ratio formula, replace d by δ for fully continuous insurance

$\boxed{\text{Know for E3 HW8}}$ For discrete whole life

22} insurance, if mortality follows the illustrative life table, then ${}_tV_x =$

$$1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x} = \frac{A_{x+t} - A_x}{1 - A_x}$$

63.5

ex] * For discrete life insurance on (52) mortality follows the illustrative life table and $i = 0.06$. Calculate ${}_{23}V_{52}$ using a) the annuity ratio formula b) the insurance ratio formula.

soln] a) $${}_tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$$ so

$${}_{23}V_{52} = 1 - \frac{\ddot{a}_{75}}{\ddot{a}_{52}} = 1 - \frac{7.2170}{12.8879}$$

$$= \boxed{0.4400}$$

If insurance benefit $= 10000 = B$ multiply ${}_tV$ by 10000 so $\boxed{4400}$

table gives $1000A_x$

b) $${}_tV_x = \frac{A_{x+t} - A_x}{1 - A_x} = \frac{A_{75} - A_{52}}{1 - A_{52}} =$$

$$\frac{.59149 - .27050}{1 - .27050} = \boxed{.4400}$$

ex] (for discrete endowment insurance, using 21)

$${}_tV_{x:\overline{n}|} = 1 - \frac{\ddot{a}_{x+t:\overline{n-t}|}}{\ddot{a}_{x:\overline{n}|}}$$

$$= \frac{A_{x+t:\overline{n-t}|} - A_{x:\overline{n}|}}{1 - A_{x:\overline{n}|}} = \frac{P_{x+t:\overline{n-t}|} - P_{x:\overline{n}|}}{P_{x+t:\overline{n-t}|} + d}$$

p234

23] Notation: calculate the 15th terminal (benefit) reserve = 15th policy value

means find ${}_{15}V$ so $t = 15$.

24]* For fully continuous whole life insurance

by 21) $${}_t\bar{V}(\bar{A}_x) = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x} = \frac{\bar{A}_{x+t} - \bar{A}_x}{1 - \bar{A}_x} = \frac{\bar{P}(\bar{A}_{x+t}) - \bar{P}(\bar{A}_x)}{\bar{P}(\bar{A}_{x+t}) + \delta}.$$

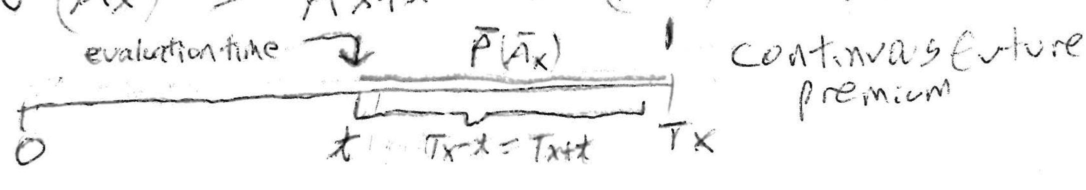
p51

25] $${}_t\bar{L}(\bar{A}_x) = v^{\overbrace{T_{x+t}}^{T_x - t}} - \bar{P}(\bar{A}_x)\,\bar{a}_{\overline{T_{x+t}}|}$$

$$= \bar{Z}_{x+t} - \bar{P}(\bar{A}_x)\,\bar{Y}_{x+t}$$

where $T_{x+t} = T_x - t$. Taking expectations

gives $${}_t\bar{V}(\bar{A}_x) = \bar{A}_{x+t} - \bar{P}(\bar{A}_x)\,\bar{a}_{x+t}.$$



64.5

Note: reserves are hard. Need to know

A, $\ddot{a}$, P

skip

8	2	2	4	$= 128$
insurance	insurance discrete continuous	premiums discrete or continuous	formula	
1 whole	A or $\bar{A}$	$\ddot{a}$ or $\bar{a}$	prospective	
n year term			retrospective	$+ 12$
n year endowment			premium difference	
n year pure endowment			paidup insurance	$= 140$
n year deferred n premiums				
n year deferred premiums for life				
n year deferred annuity				
h pay whole life				

	insurance	annuity
fully discrete	d	d
full continuous	c	c
semi continuous	d	c
	c	d

$+$ 4

3
annuity ratio
insurance ratio
premium ratio

for discrete and continuous whole & endowment insurance

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p 10.2 26] Let B be the benefit paid. Then

$${}_{t+1}V = ({}_{t}V + P)(1+i) - q_{x+t}\,(\overset{\text{alt}}{B} - {}_{t+1}V)$$

$B = 1$ for 1 unit payment is the usual case.

27] Let π_t be the premium paid at time $t = 0, 1, \ldots$ and let b_k be the benefit paid at time k if death occurs in the kth year of the policy $k = 1, 2, \ldots$. Then

$$({}_{k-1}V + \pi_{k-1})(1+i) = q_{x+k-1}(b_k - {}_{k}V) + {}_{k}V$$

§7.3 28) p276 Suppose premiums are paid mthly. Let t be integer valued. So ${}_tV^{(m)}$ is measured at annual intervals.

$m=4$ quarterly $m=12$ monthly $m=52$ weekly

29) p252 i) discrete whole life

$$ {}_tV_x^{(m)} = A_{x+t} - P_x^{(m)} \ddot{a}_{x+t}^{(m)} $$

Now suppose $t < n$.

ii) n year term

$$ {}_tV_{x:\overline{n}|}^{(m)} = A^{\,1}_{x+t:\overline{n-t}|} - P^{1\,(m)}_{x:\overline{n}|} \ddot{a}^{(m)}_{x+t:\overline{n-t}|} $$

iii) n year pure endowment

$$ {}_tV^{(m)}_{x:\overset{1}{\overline{n}|}} = A_{x+t:\overset{1}{\overline{n-t}|}} - P^{(m)}_{x:\overset{1}{\overline{n}|}} \ddot{a}^{(m)}_{x+t:\overline{n-t}|} $$

iv) n year endowment

$$ {}_tV_{x:\overline{n}|}^{(m)} = A_{x+t:\overline{n-t}|} - P^{(m)}_{x:\overline{n}|} \ddot{a}^{(m)}_{x+t:\overline{n-t}|} $$

v) n year deferred immediate annuity

$$ {}_tV^{(m)}({}_{n|}a_x) = {}_{n-t|}a_{x+t} - P^{(m)}({}_{n|}a_x)\, \ddot{a}^{(m)}_{x+t:\overline{n-t}|} $$

30] mthly premiums immediate payment (65.5)
(continuous) whole life insurance

$${}_tV^{(m)}(\bar{A}_x) = \bar{A}_{x+t} - P^{(m)}(\bar{A}_x)\,\ddot{a}^{(m)}_{x+t}$$

$\S 6.4, 2$ / 7.2 31] The ${}_tV$ terminal reserve = policy value was for benefits only.

Suppose we now also include expense factors. Then ${}_tV^E$ =

$$\text{(APV } \underbrace{\text{of future benefits and expenses}}^{\text{liabilities}}\text{)}$$

$-$ (APV of future gross premiums)

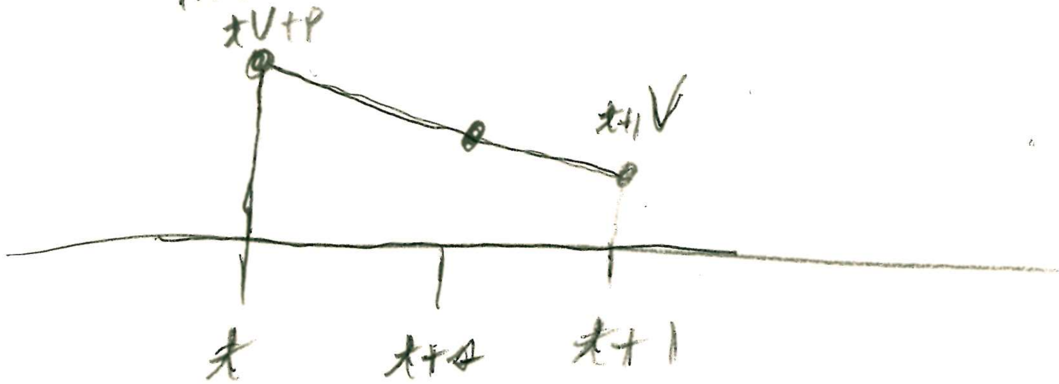
are expense-augmented reserves = gross premium reserves, A gross premium G takes into account the expenses. Terminal (Benefit) reserves ${}_tV$ use benefit premiums = net premiums P. The notation ${}_tV = {}_tV^N$ is sometimes used.

§11.2 32] p275 Terminal reserve ${}_{t+s}V$ (402 66F)

when t is an integer and $0 \le s \le 1$:

$${}_{t+s}V \approx \underbrace{({}_tV + P)}_{\text{reserve just after premium is paid at time } t}(1-s) + ({}_{t+1}V)s,$$

linear interpolation of ${}_tV + P$ and ${}_{t+1}V$



where premiums are paid at integers k, including $k = t$.

(Continuous payment reserves are defined for $t > 0$.)

Note: ${}_0V = 0$ if premiums are from the equivalence principle.

ex] Suppose ${}_9V = 100$, ${}_{10}V = 105.2632$.

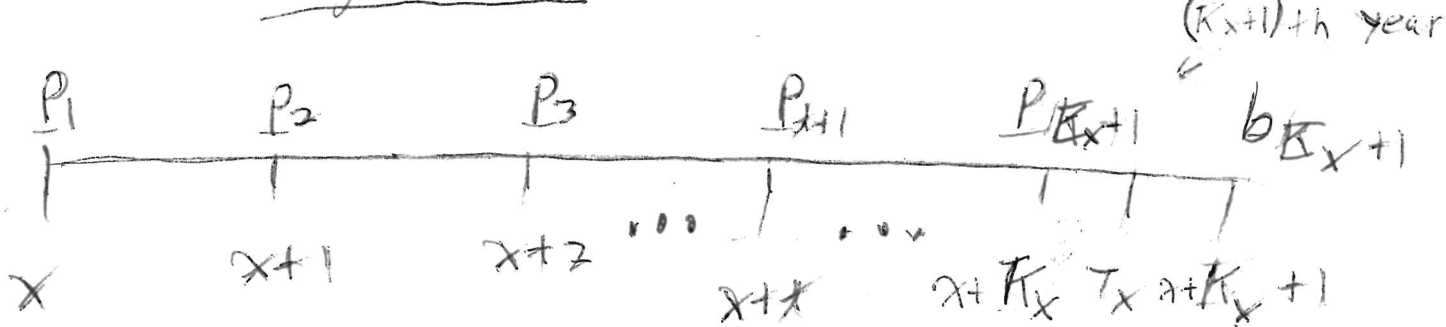
Approximate ${}_{9.5}V$ if $P = 20$.

soln $\approx ({}_9V + P)\frac{1}{2} + ({}_{10}V)\frac{1}{2} =$

$$\frac{100 + 20 + 105.2632}{2} = \boxed{112.6316}$$

§11.3 33] p276-7 Let the benefit payment for failure in the kth year, payable at the end of the year, be b_k. (66.5)

Let the benefit premium, payable at the beginning of the kth year, be P_k.



At time $t=0$,

The APV of the benefit is

$$\sum_{k=0}^{\infty} b_{k+1} v^{k+1} P(K_x = k) = E\left(b_{K_x+1} v^{K_x+1}\right)$$

$$= \sum_{k=0}^{\infty} b_{k+1} v^{k+1} {}_kp_x\, q_{x+k},$$ while the

APV of the premiums is $\sum_{k=0}^{\infty} P_{k+1} v^k {}_kp_x$

with ${}_tV = \sum_{k=0}^{\infty} b_{t+k+1} v^{k+1} {}_kp_{x+t}\, q_{x+t+k} -$

$\sum_{k=0}^{\infty} P_{t+k+1} v^k {}_kp_{x+t}$.

34] [278] For a fully continuous whole life model with payment of benefit amount b_r at time of death r and benefit payment at rate $\bar{P}(r)$ at time r, given $T_x > t$,

$$_t\bar{V} = \int_0^\infty b_{t+s}\, e^{-\delta s}\, {}_sp_{x+t}\, \mu_{x+t+s}\, ds$$

$$- \int_0^\infty \bar{P}(t+s)\, e^{-\delta s}\, {}_sp_{x+t}\, ds.$$

35] Know Suppose $\mu_{x+t} \equiv \mu$, $t \geq 0$,

$$b_{t+s} = J\, e^{\theta(t+s)}, \quad \bar{P}(t+s) = \Pi_0\, e^{\gamma(t+s)}$$

$t, s \geq 0$, where Π_0 is the premium at time t $(s = 0)$. Then $T_{x+t} \sim EXP(\mu)$, $t \geq 0$.

So ${}_sp_{x+t} = e^{-\mu s}$, $s \geq 0$. To find Π_0,

use ${}_0\bar{V} = 0$ or equate the 2 APVs at $t=0$:

$\int_0^\infty \Pi_0\, e^{\gamma s}\, e^{-\delta s}\, {}_sp_{x+t}\, ds = \int_0^\infty b_s\, e^{-\delta s}\, {}_sp_{x+t}\, \mu_{x+t+s}\, ds$ or

$$\Pi_0 \int_0^\infty e^{\gamma s}\, e^{-\delta s}\, e^{-\mu s}\, ds = \Pi_0 \int_0^\infty e^{-s(\mu+\delta-\gamma)}\, ds$$

$$= \frac{\Pi_0}{\mu+\delta-\gamma} = J \int_0^\infty e^{\theta s}\, e^{-\delta s}\, e^{-\mu s}\, \mu\, ds =$$

$$J\mu \int_0^\infty e^{-s(\mu+\delta-\theta)}\,ds = \frac{J\mu}{\mu+\delta-\theta}.$$

So
$$\boxed{\pi_0 = \frac{(\mu+\delta-\gamma)\,J\mu}{\mu+\delta-\theta}}.$$
Then

$${}_t\bar{V} = \int_0^\infty b_{t+s}\, e^{-\delta s}\, {}_sP_{x+t}\,\mu\, ds - \int \text{''} \quad \text{''}$$

$$= \int_0^\infty J e^{\theta(t+s)} e^{-\delta s} e^{-\mu s} \mu\, ds - \int_0^\infty \bar{P}(t+s)\, e^{-\delta s}\, {}_sP_{x+t}\, ds$$

$$= J\mu e^{\theta t} \int_0^\infty e^{-s(\mu+\delta-\theta)}\, ds - \int_0^\infty \pi_0 e^{\gamma(t+s)} e^{-\delta s} e^{-\mu s}\, ds$$

$$= J\mu e^{\theta t} \int_0^\infty e^{-s(\mu+\delta-\theta)}\, ds - \pi_0 e^{\gamma t} \int_0^\infty e^{-s(\mu+\delta-\gamma)}\, ds$$

$$= \boxed{{}_t\bar{V} = \frac{J\mu e^{\theta t}}{\mu+\delta-\theta} - \frac{\pi_0 e^{\gamma t}}{\mu+\delta-\gamma}}$$

$\mu+\delta-\theta>0,\ \mu+\delta-\gamma>0$

36] $\bar{P}(t+s) = \pi_0 e^{\gamma(t+s)}$ is often written as "the annual premium rate is $\pi_0 e^{\gamma t}$ for all t." The benefit

$b_{t+s} = J e^{\theta(t+s)}$ is often written as

" the benefit is $J e^{\theta t}$ if death occurs at time t." If $\gamma = 0$, then

$$\bar{P}(t+s) \equiv \Pi_0 = \frac{J\mu(\mu+\delta)}{\mu+\delta-\theta} \quad \text{for } t, s \geq 0.$$

If $\theta = 0$, then $b_{t+s} \equiv J$ for $t, s \geq 0$.

ex] suppose $\mu_{x+t} \equiv \mu$, $\theta = 0$, $\gamma = 0$ and $J = 1$.

Then $\Pi_0 = \dfrac{\mu(\mu+\delta)}{\mu+\delta} = \mu$ and

$$ {}_t\bar{V} = \frac{\mu}{\mu+\delta} - \frac{\mu}{\mu+\delta} = 0 \stackrel{E}{=} {}_t\bar{V}(\bar{A}_x).$$

If $J \neq 1$, $\Pi_0 = J\mu$ and ${}_t\bar{V} = 0$.