More Topics from ch 7–10, Markov Chains (App A 10.7)

1] p358 §10.5 ch 9 Assume time and cause of decrement are ind.
Suppose a continuous whole life insurance pays benefit $b_t^{(j)}$ if decrement j occurs at time t. Then the single benefit premium (net single premium, buy insurance with 1 premium at time $t=0$)

is $\bar{A} = E(\bar{Z}) = \sum_{j=1}^{m} \int_0^\infty b_t^{(j)} e^{-\delta t} {}_tp_x^{(\tau)} \mu_{x+t}^{(j)} dt$

$= \sum_{j=1}^{m} \bar{A}^{(j)} = \text{APV}.$

Often $b_t^{(j)} = b_j$, a constant that does not depend on t.

$$E[\bar{Z}^2] = \sum_{j=1}^{m} \int_0^\infty \left[b_t^{(j)}\right]^2 e^{-2\delta t} {}_tp_x^{(\tau)} \mu_{x+t}^{(j)} dt$$
$$= \sum_{j=1}^{m} {}^2\bar{A}^{(j)}.$$

Here $\bar{Z}$ is the benefit random variable for the insurance, and there are m decrements.

If $\mu_{x+t}^{(j)} \equiv \mu_j$, then $\mu^{(\tau)} = \sum_{j=1}^{m} \mu_j$

and ${}_tp_x^{(\tau)} = e^{-\mu^{(\tau)} t}$. Suppose $b_t^{(j)} = b_j$, then the single benefit premium =

$$E(\bar{Z}) = \sum_{j=1}^{m} b_j \int_0^\infty e^{-\delta t} e^{-\mu^{(\tau)} t} \mu^{(j)} \, dt$$

$$= \sum_{j=1}^{m} \frac{b_j \mu^{(j)}}{\mu^{(\tau)} + \delta} = \sum_{j=1}^{m} \bar{A}^{(j)}, \text{ and}$$

$$E(\bar{Z}^2) = \sum_{j=1}^{m} b_j^2 \int_0^\infty e^{-2\delta t} e^{-\mu^{(\tau)} t} \mu^{(j)} \, dt$$

$$= \sum_{j=1}^{m} \frac{b_j \mu^{(j)}}{\mu^{(\tau)} + 2\delta} = \sum_{j=1}^{m} {}^2\bar{A}^{(j)}.$$

If $b_j \equiv b$ then $E\bar{Z} = b \frac{\mu^{(\tau)}}{\mu^{(\tau)}+\delta}$ and $E\bar{Z}^2 = b^2 \frac{\mu^{(\tau)}}{\mu^{(\tau)}+2\delta}$.

ex] MLC 5 A whole life policy provides that upon accidental death as a passenger on an airplane, a benefit of 1,000,000 $(j=1)$ will be paid. If death occurs from other accidental causes, a death benefit of 500000 $(j=2)$ will be paid. If death occurs from a cause other than an accident, a death benefit of 250000 will be paid $(j=3)$. Death benefits are $\underbrace{\text{payable at the moment of death}}_{\text{key words for continuous}}$. $\mu^{(1)} = \frac{1}{2000000}$

70.5

$\mu^{(2)} = \frac{1}{250000}$, $\mu^{(3)} = \frac{1}{10000}$, $\delta = 0.06$.

Find the single benefit premium.

Soln] $APV = \dfrac{\sum_{j=1}^{m} b_j \, \mu^{(j)}}{\mu^{(\tau)} + \delta} =$

$$\frac{\frac{1000000}{2000000} + \frac{500000}{250000} + \frac{250000}{10000}}{\left(\frac{1}{2000000} + \frac{1}{250000} + \frac{1}{10000}\right) + .06}$$

$$= \frac{0.5 + 2 + 25}{\underbrace{0.0001045}_{\mu^{(\tau)}} + 0.06} = \frac{27.5}{0.0601045} = \boxed{457.5365}$$

see HW9 #3

Note if $T \sim EXP\left(\mu = \frac{1}{10000}\right)$, $E(T) = 10000$.

So numbers are not realistic.

p(H.)

p 361 Assume time $\perp$ cause of decrement.

2] If there are m decrements and discrete whole life insurance pays benefit b_j if decrement j occurs in the k th year for $k = 1, 2, \ldots$ then

$$\bar{A}^{(j)} = b_j \sum_{K=0}^{\infty} v^{K+1} \, {}_K p_x^{(\tau)} \, q_{x+k}^{(j)} \quad \text{and.}$$

$K_x = 0$ in 1st year

$$APV = \text{net single premium} = \sum_{j=1}^{m} \bar{A}^{(j)}!$$

See HW9 #1

ex] * p 361-2 A five year bond, issued at time 0, has 3 decrements 1) default 2) call (prepayment) 3) maturity,

K	$q_K^{(1)}$	$q_K^{(2)}$	$q_K^{(3)}$	${}_K p_0^{(\tau)}$	v^{K+1}
0	.02	.03	0.0	1	.943
1	.02	.04	0.0	.95	.890
2	.02	.05	0.0	.893	.840
3	.02	.06	0.0	.83	.792
4	.02	0.0	.98	.764	.747

everyone "fails" in year 5 = maturity

$.95 = 1(1-.05)$

$.893 = .95(1-.06)$

$.83 = .893(1-.07)$

$.764 = .83(1-.08)$

A guarantor will pay 1000 in event of default and nothing, otherwise. Find the APV of the guarantor's contingent payment contract, if $i = 0.06$.

Soln] time $= 0 \rightarrow x = 0$

$$APV = \bar{A}^{(1)} = 1000 \sum_{K=0}^{4} v^{K+1} \, {}_K p_0^{(\tau)} \, q_{0+K}^{(1)}$$

$$= 1000[.943(1).02 + .89(.95).02 + .84(.893).02 + .792(.83).02 + .747(.764).02] = \boxed{75.3338}$$

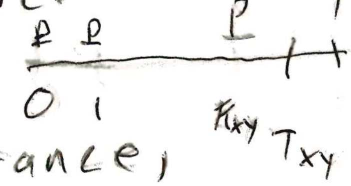
Ch 10: policy values M402 72

§12.4.3 Premiums and reserves for (xy) and $(\overline{xy})$.

3] p304 The annual benefit premium for a discrete whole life joint life insurance is $P_{xy} = \dfrac{A_{xy}}{\ddot{a}_{xy}}$ where benefit

premium not survival function

is paid at the end of the year of 1st failure and premiums stop after 1st failure.



For the last survivor insurance,

$$P_{\overline{xy}} = \frac{A_{\overline{xy}}}{\ddot{a}_{\overline{xy}}}$$

where premiums continue until the 2nd failure and the insurance is paid at the end of the year of the 2nd failure.

4] Know For fully continuous whole life unit benefit insurance (Premiums paid continuously until 1st death for (xy) and 2nd for $\overline{xy}$

$$\bar{P}(\bar{A}_{xy}) = \frac{\bar{A}_{xy}}{\bar{a}_{xy}} \text{ and } \bar{P}(\bar{A}_{\overline{xy}}) = \frac{\bar{A}_{\overline{xy}}}{\bar{a}_{\overline{xy}}} = \frac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_x + \bar{a}_y - \bar{a}_{xy}}$$

7L.5

5]* Variant: Fully continuous whole life insurance of 1 on last survivor of (x) and (y), but premiums are payable (continuously) until first death.

Then premium $\bar{P} = \dfrac{\bar{A}_{\overline{xy}}}{\bar{a}_{xy}} = \dfrac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_{xy}}$.

see HW9 #4

6] know If $T_x \sim EXP(\mu_1)$ ⫫ $T_y \sim EXP(\mu_2)$,

then $T_{xy} \sim EXP(\mu_1+\mu_2)$, $\bar{A}_w \overset{E}{=} \dfrac{\mu}{\mu+\delta}$, $\bar{a}_w \overset{E}{=} \dfrac{1}{\mu+\delta}$.

variant: common shock model where

$T_{xy} \sim EXP(\mu_1^* + \mu_2^* + \lambda)$

$T_x \sim EXP(\mu_x = \mu_1^* + \lambda)$, $T_y \sim EXP(\mu_y = \mu_2^* + \lambda)$.

7] If $(w) = (xy)$ or $(w) = (\overline{xy})$,

premiums are computed in the usual way:

APV insurance $= A_w =$ APV Premiums $= P\, a_w$

so $P = \dfrac{A_w}{a_w}$.

8] A variant is when insurance is paid after 2nd death but premiums only up to 1st death. Then

$$P = \frac{A_{\overline{xy}}}{a_{xy}} \quad \begin{matrix} \leftarrow \text{last survivor} \\ \leftarrow \text{joint life} \end{matrix}$$

see 5].

9] It is not true that $P_x + P_y = P_{xy} + P_{\overline{xy}}$ where P indicates premium (the formula is true for survival functions).

10) P305 Treat terminal reserves (policy values =) for $(w) = (xy)$ like the reserve for a single life status (w) provided the status $(w) = (xy)$ has not yet failed at time t. Note that subscript $w+t = x+t : y+t$.

ex] ${}_tV_{xy} = A_{x+t:y+t} - P_{xy}\, \ddot{a}_{x+t:y+t}$

11] P305 Terminal reserves (= Policy values) for $(\overline{xy})$ depend on whether

73.5

i) both (x) and (y) still survive

ii) (x) survives but (y) failed or

iii) (x) failed but (y) survives

at time t.

If i) holds, then the premium is as for a single status $(w) = (\overline{xy})$ where subscript $w+t = \overline{x+t : y+t}$.

So ${}_tV_{\overline{xy}} = A_{\overline{x+t : y+t}} - P_{\overline{xy}} \, \ddot{a}_{\overline{x+t : y+t}}$

Ch 10
§ 9.5 – 9.6

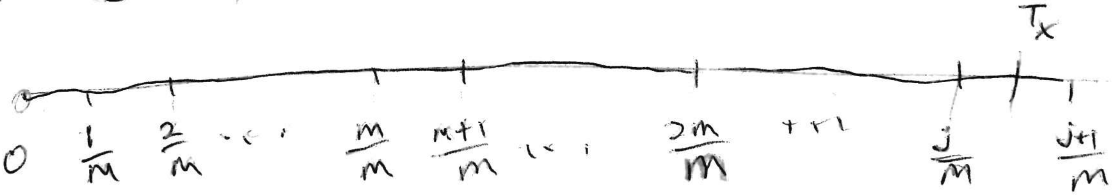
p. 2?2

12 § 9.5 Funding schemes with mthly payments:

$P^{(m)}$ is the annual funding rate and

$\frac{P^{(m)}}{m}$ is the premium (actual funding payment) paid every $\frac{1}{m}$th of a year

13]



M402 74

For discrete insurance the payment is made at the end of the interval of failure. (interval is $\frac{1}{m}$th of a year, eg end of day, week, month, quarter, 6months for $m = 365, 52, 12, 4, 2$).

13) p222 Suppose there is mthly funding, discrete insurance. (Did reserves earlier)

i) discrete whole life $P^{(m)} = \dfrac{A_x}{\ddot{a}^{(m)}}$

ii) n year term $P^{(m)}_{\overset{1}{x}:\overline{n}|} = \dfrac{A^{1}_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}}$

iii) n year endowment $P^{(m)}_{x:\overline{n}|} = \dfrac{A_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}}$

iv) n year deferred immediate annuity

$$P^{(m)}(n|a_x) = \frac{n|a_x}{\ddot{a}^{(m)}_{x:\overline{n}|}}$$

15] Suppose insurance payment is made immediately after death (semi continuous insurance)

i) whole life $P^{(m)}(\bar{A}_x) = \dfrac{\bar{A}_x}{\ddot{a}_x^{(m)}}$

ii) n year term $P^{(m)}(\bar{A}^{1}_{x:\overline{n}|}) = \dfrac{\bar{A}^{1}_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}}$

iii) n year endowment $P^{(m)}(\bar{A}_{x:\overline{n}|}) = \dfrac{\bar{A}_{x:\overline{n}|}}{\ddot{a}^{(m)}_{x:\overline{n}|}}$

§6.4

15] 9.6 A gross premium G takes into account expenses. G is also called a gross annual premium, contract premium, or expense augmented premium.

16] *Equivalence principle

$E({}_0L_e) = 0 =$ APV(benefits + expenses) − APV(gross premiums)

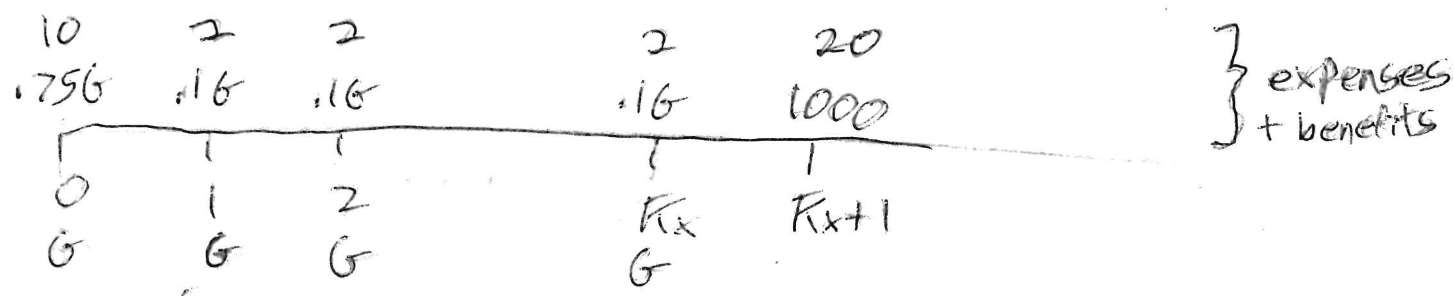
where ${}_0L = {}_0L_e$ is the loss random variable at issue.

18) For discrete insurance

$$APV(\text{gross premiums}) = G\,\ddot{a}_x$$

ex 9.10] Discrete whole life insurance

expenses: 75% of 1st premium and 10% of all premiums thereafter, \$10 at beginning of 1st year, \$2 at beginning of each year thereafter, \$20 settlement expense, \$1000 benefit



$APV(\text{expenses + benefits}) = 10 + 2a_x + .75G$

$+ .1G\,a_x + 1020\,A_x = APV(\text{premiums}) = G\,\ddot{a}_x$.

so $G(\ddot{a}_x - .75 - .1a_x) = 1020A_x + 10 + 2a_x$

$$G = \frac{1020A_x + 10 + 2a_x}{\ddot{a}_x - .75 - 0.1a_x}$$

Suppose $(x) = (40)$ follows the illustrative life table. Using $\ddot{a}_x = a_x + 1$, ← certain payment at $t=0$

table gives $1000A_x$

$\ddot{a}_{40} = 14.8166$, $a_{40} = 13.8166$, $A_x = 0.16132$.

$$\text{So } G = \frac{1020(0.16132) + 10 + 2(13.8166)}{14.8166 - 0.75 - 0.1(13.8166)}$$

$$= \frac{202.1796}{12.68494} = 15.9386$$

ex] Variant Given G and death in kth year, k small, find (observed value of)

${}_0L = {}_0L_e$ = APV(benefits + expenses) − APV(premiums) for person who died in kth year.

ex] MLC 198 For a fully discrete whole life insurance of 1000 on (60), you are given

i) The expenses, payable at the beginning of the year, are

Expense Type	First Year	Renewal Years
% of premium	20%	6%
per policy	8	2

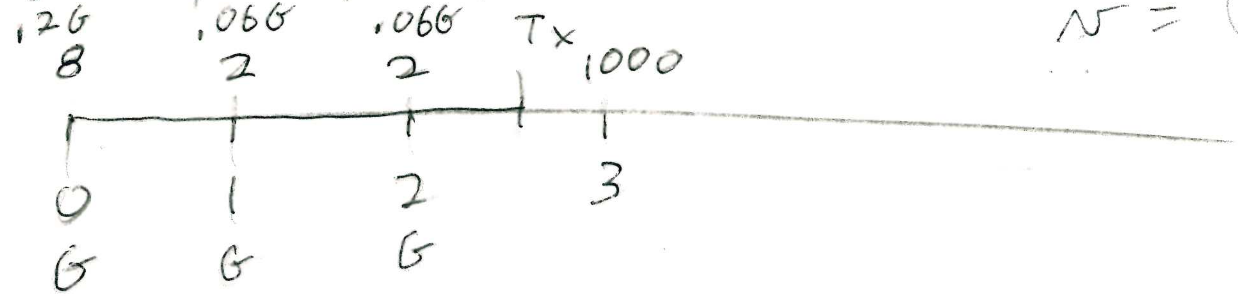
ii) the gross premium is 41.20

iii) $i = 0.05$, iv) ${}_0L$ is the present value of the loss random variable at issue. 402 36

Calculate $_0L$ if the insured dies in the

3rd policy year

$$v = (1+i)^{-1}$$



QMD 15

$$APV(\text{premiums}) = G + Gv + Gv^2$$

$$APV(\text{expenses} + \text{benefits}) = .2G + 8 + (.06G + 2)v$$

$$+ (.06G+2)v^2 + 1000v^3$$

observed loss for the insured =

$$_0L = APV(\text{expenses} + \text{benefits}) - APV(\text{Premiums})$$

$$= 16.24 + (4.472)(1.05)^{-1} + 4.472(1.05)^{-2} + 1000(1.05)^{-3}$$

$$- 41.2(1 + (1.05)^{-1} + (1.05)^{-2})$$

$$= 888.3929 - 117.8077 = \boxed{770.5852}$$

69.5 9.6

end ch 6 material

HW 9 # 5 is the

same except $i = 0.04$

19]* § 14.2 §7.2 Asset Shares 76.5

or 1 or 2

$$ {}_kAS = \frac{\left[{}_{k-1}AS + G\left(1-c_{k-1}\right) - e_{k-1}\right](1+i) - b_k\, q^{(d)}_{x+k-1} - {}_kCV\, q^{(w)}_{x+k-1}}{1 - q^{(d)}_{x+k-1} - q^{(w)}_{x+k-1}} $$

Given all of the unknowns except 1, usually ${}_kAS$ or i, find the unknown.

See MLC 236 = HW9 #6, 242, (also 235 and 244)

20] ${}_kAS$ is the asset share at the end of year k

G is the annual contract = gross premium

c_k is the proportion of the premium payable as an expense at time k starting at $k=0$

~~$r_k = c_{k-1}$~~

e_k is the per policy expense at time k (including per 1000 expense multiplied by face amount over 1000) starting at $k=0$

$b_k = b_k^{(1)}$ is the face amount

402 77

$q^{(d)} = q^{(1)}$ = death probability

$q^{(w)} = q^{(2)}$ = withdrawal probability

${}_kCV$ is the cash value at time k: the amount the policy holder gets if she surrenders the policy at time k, ~~(text uses $b_k^{(2)}$)~~.

21) An asset share measures the accumulation of cash income per surviving policy. A (reserve=Policy value) policy value ${}_kV$ considers death benefits but ignores expenses and withdrawals. An asset share ${}_kAS$ considers death benefits, surrender benefits, withdrawals, and expenses.

22) Reasonably priced policies are profitable, so the final asset share at maturity is usually positive.

23 Formula is for a fully discrete insurance

so premiums are paid at the beginning 77.5
of the year and benefits at the end
of the year.

2\) * Usually assume ${}_0AS = 0$.

see HW9

ex] MLC 242 For a wholly discrete insurance of 10000 on (x), you are given

i) ${}_{10}AS = 1600$ is the asset share at the end of year 10.

ii) $G = 200$ is the gross premium. could ignore verbal description

iii) ${}_{11}CV = 1700$ is the cash value at the end of year 11.

iv) $c_{10} = 0.04$ is the fraction of the gross premium paid at time 10 for expenses.

v) $e_{10} = 70$ is the amount per policy expense paid at time 10.

vi) Death and withdrawal are the only decrements.

vii) $q^{(d)}_{x+10} = 0.02$

viii) $q^{(w)}_{x+10} = 0.18$

ix) $i = 0.05$.

Calculate ${}_{11}AS$, the asset share at the end of year 11.

M402 38

soln] $b_K = 10000$ and

$$_{11}AS = \frac{[{}_{10}AS + G(1-c_{10}) - e_{10}](1+i) - b_K\, q^{(d)}_{x+10} - {}_{11}CV\, q^{(w)}_{x+10}}{1 - q^{(d)}_{x+10} - q^{(w)}_{x+10}}$$

$$= \frac{[1600 + 200(1-.04) - 70]1.05 - 10000(.02) - 1700(.18)}{1 - .02 - .18}$$

$$= \frac{1302.1}{0.8} = \boxed{1627.63}$$

skip

38]