M402 6479

ex) MLC 249 for (x) & (y)

(x) satisfies UDD

(y) has constant force of mortality 0.02

$q^{1}_{xy} = 0.125$. Find q_x.

Soln $q^{1}_{xy} = \int_0^1 {}_tp_{xy}\, \mu_{x+t}\, dt =$

$\int_0^1 {}_tp_x\, {}_tp_y\, \mu_{x+t}\, dt \approx \int_0^1 q_x\, e^{-0.02t}\, dt$

UDD ${}_tp_x \mu_{x+t} = q_x$ M40(73) (Elrev 402 11)

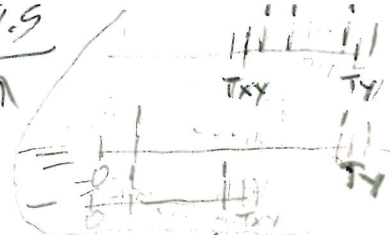
$= q_x \left(\frac{e^{-0.02t}}{-0.02} \Big|_0^1 \right) = q_x (-50)(e^{-0.02} - 1) = \ldots q_x$

So $q_x \approx \frac{0.125}{0.9868} = \boxed{0.1413}$

§12.4

25) p 306 §10.3 A <u>reversionary annuity</u> is one that pays only after one of the lives has failed, and then for as long as the other continues to survive. If annuity payment is such that payment is made to

(y) after (x) dies then



i) $a_{x|y} = a_y - a_{xy}$ discrete whole life

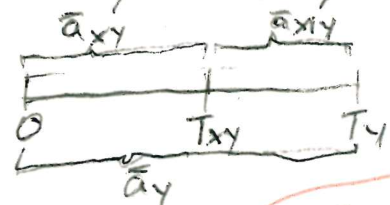
ii) $a_{x|y:\overline{n}|} = a_{y:\overline{n}|} - a_{xy:\overline{n}|}$ discrete term

(payment is made n years at most where (x) has failed but (y) has not)

pays until (y) fails

pays until (xy) fails

iii) * continuous whole $\bar{a}_{x|y} = \bar{a}_y - \bar{a}_{xy}$



continuous term

iv) $\bar{a}_{x|y:\overline{n}|} = \bar{a}_{y:\overline{n}|} - \bar{a}_{xy:\overline{n}|}$

For $x|y$ contract ends if (y) dies payment begins if (x) dies.

26] Premiums end after failure of joint life status.

$$P(a_{x|y}) = \frac{a_{x|y}}{\ddot{a}_{xy}} = \frac{a_y - a_{xy}}{\ddot{a}_{xy}}$$ funded by annual premiums

$$\bar{P}(\bar{a}_{x|y}) = \frac{\bar{a}_{x|y}}{\bar{a}_{xy}} = \frac{\bar{a}_y - \bar{a}_{xy}}{\bar{a}_{xy}}$$ funded by continuous premiums

27] If both (x) and (y) are alive, then $_tV(a_{x|y}) = a_{x+t|y+t} - P(a_{x|y})\,\ddot{a}_{x+t:y+t}$

M402 65-80

$${}_t\bar{V}(\bar{a}_{x|y}) = \bar{a}_{x+t:y+t} - \bar{P}(\bar{a}_{x|y})\,\bar{a}_{x+t:y+t}.$$

If only (y) is alive, then

${}_tV(a_{x|y}) = a_{y+t}$ and ${}_t\bar{V}(\bar{a}_{x|y}) = \bar{a}_{y+t}$.

If only (x) is alive, then ${}_tV = 0$ and ${}_t\bar{V} = 0$, the contract is expired.

28) If payment is made to (x) only after (y) dies, use formulas like

$$\bar{a}_{y|x} = \bar{a}_x - \bar{a}_{xy}, \quad P(\bar{a}_{y|x}) = \frac{\bar{a}_{y|x}}{\bar{a}_{xy}} = \frac{\bar{a}_x - \bar{a}_{xy}}{\bar{a}_{xy}}.$$

§ 8.10

Markov Chains: Insurance, Premiums, ~~Reserves~~ Policy Values

§ 10.5

29) Insurance

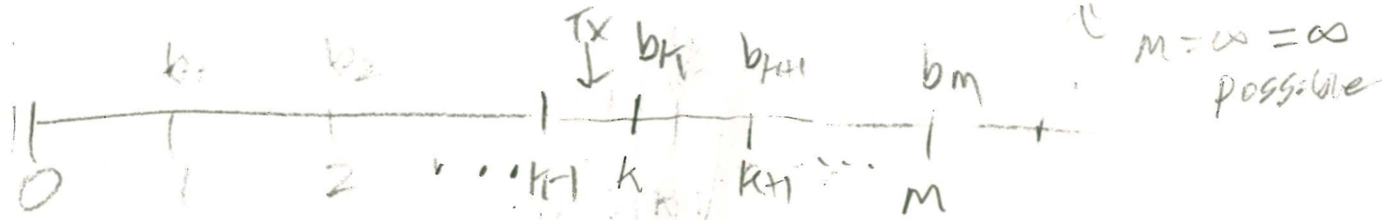
a) For single life mortality, the benefit is payable upon transition from alive to dead.

b) For multiple decrements, there could

be different benefits paid for different types of death (accident, not an accident) or a benefit for disabled and a benefit for death.

(~~30~~ uses $p_k = {}_{k-1|}q_x$ In M401 used $p_k = {}_{k|}q_x$.) Skip

~~30~~) Review of discrete insurance APV. Suppose insurance pays benefit b_k if $\lfloor T_x \rfloor = k-1$ at time k with probability p_k, $k = 1, 2, \dots, m$.



If Z is the present value random variable, then the possible values of Z are the discounted benefits.

$Z =$	$b_1 v$	$b_2 v^2$ ...	$b_m v^m$
prob	p_1	p_2	p_m

$p_k = P(\lfloor T_x \rfloor = k-1)$
$= P(K_x = k-1)$
$= {}_{k-1|}q_x$

Here $\sum_{k=1}^{m} p_k = 1$.

So $E(Z) = APV = \sum_{k=1}^{m} b_k v^k p_k$ $\left(= \sum_{k=0}^{m-1} b_{k+1} v^{k+1} \underbrace{{}_{k|}q_x}_{p_k \text{ in M401}} \right)$

Double decrement has b_{k1}, b_{k2}, $p_{kj} = P(\lfloor T_x \rfloor = k, \text{decrement} = j)$

Note that each summand is a triple product of i) the benefit, ii) the discount factor and iii) the probability that the benefit is paid.

30] The APV of a set of cashflows using a Markov Chain (MC):

The APV is a sum of a triple product of i) the cashflow at time k ii) v^k, and iii) the probability that the cashflow occurs (computed using the markov chain).

ex] MLC 89

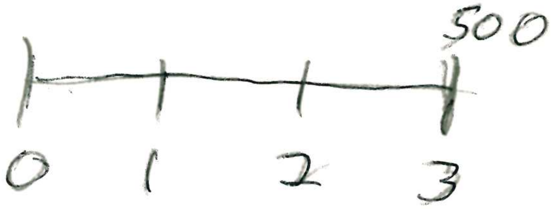
$$\mathbb{P} = Q = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0.75 & 0 & 0 & 0.25 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

E3d15

At time 0, subject is in state 1. A payment of 500 will be made at the end of 3 years if the subject is in state 1. M4 2 81.5

Find the APV of this insurance if $v = 0.9$

Soln



$APV = (\text{prob})\, 500 v^3$

If $\underline{\pi}_3 = (a \;\; b \;\; c \;\; d)$, then $\text{prob} = a$.

$\underline{\pi}_3 = \underline{\pi}_0 \, Q^3 = [1 \;\; 0 \;\; 0 \;\; 0]\, Q\, Q^2 = [.2 \;\; .8 \;\; 0 \;\; 0]\, QQ$

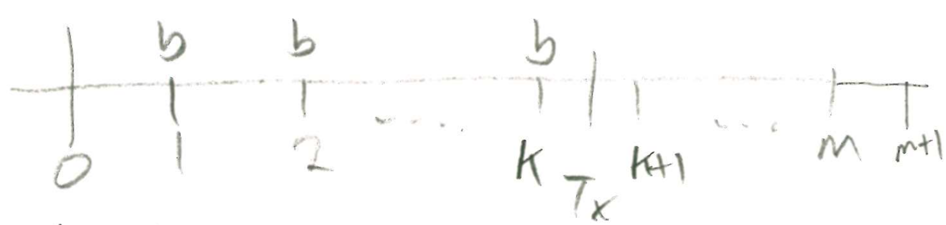
$= [.44 \;\; .16 \;\; .4 \;\; 0]\, Q = (.468 \;\; .352 \;\; .08 \;\; d)$

$(.2)^2 + .5(.8)$ $\quad .2(.8)$ $\quad .8(.5)$

1st entry $= a = .44(.2) + .16(.5) + .4(.75) = .468$

So $APV = 0.468(500)(0.9)^3 = \boxed{170.586}$

61} Review of discrete annuities Stop



Let Y be the present value random variable.

Possible values of Y are

0 if $\lfloor T_x \rfloor = 0$ and

$b\, a_{\overline{k}|}$ if $\lfloor T_x \rfloor = k$, $k = 1, 2, \ldots, m$.

$m = \infty$ possible

$$b\, a_{\overline{k}|} = b v + b v^2 + \cdots + b v^k = b\,(v + v^2 + \cdots + v^k)$$

Y	0	$b\,a_{\overline{1}|}$	$b\,a_{\overline{2}|}$	...	$b\,a_{\overline{m}|}$
prob	p_0	p_1	p_2		p_m

So $APV = E(Y) = 0 + \sum_{k=1}^{m} p_k\, b\, a_{\overline{k}|}$,

$p_k = P(\lfloor T_x \rfloor = k) = P(K_x = k) = {}_{k|}q_x$,

where $\sum_{k=0}^{m} p_k = 1$. Note! in 59], used $p_k = P(\lfloor T_x \rfloor = k-1)$.

SKIP

31] For a Markov Chain, the APV is annuity: similar to an annuity, but the number of values k where the annuity can be paid are small, often 2 or 3; and the p_k are found using a Markov Chain.

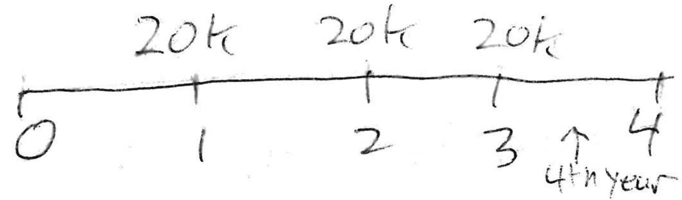
ex)

time	state vector			
1	(0.7	0.2	0.1)	$= \underline{\pi}_1$
2	(0.59	0.2	0.21)	$= \underline{\pi}_2$
3	(0.513	0.178	0.309)	$= \underset{\sim}{\pi}_3$

Suppose the states are 1) active, 2) disabled, 3) dead with

$$Q = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.5 & 0.3 & 0.2 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

"Insurance" at time 0 is sold to people classified as active. The insurance is a 4 year contract. Suppose the person gets 20000 at the beginning of the year if disabled.



No benefit is paid at the beginning of the 5th year ($t=4$) since the contract is a 4 year contract. Find the APV of the disability contract if $i = .05$.

Soln For each state vector, the 2nd component is the probability of being disabled. So

$$APV = 20000\ (0.2v + 0.2v^2 + 0.178v^3)$$

$$= 20000\left[0.2(1.05)^{-1} + 0.2(1.05)^{-2} + 0.178(1.05)^{-3}\right]$$

$$= \boxed{10512.90}$$

($Y = 0$ with prob $1 - .2 - .2 - .178$)

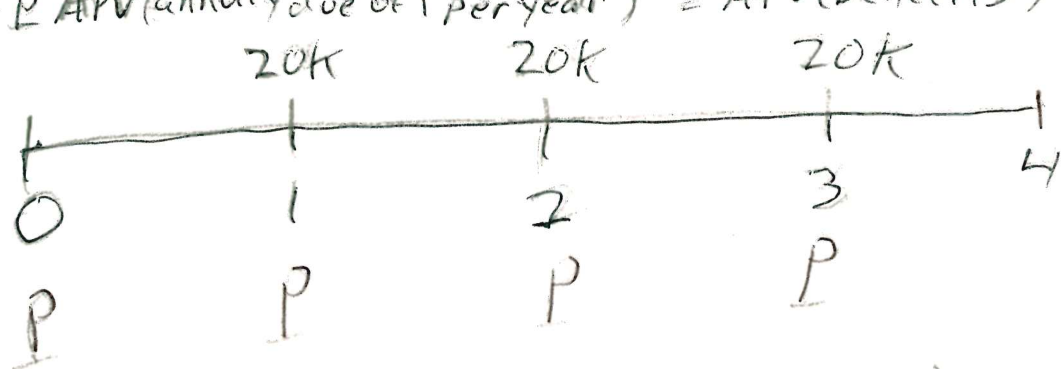
Y	$20k\,v$	$20k\,v^2$	$20k\,v^3$
K	1	2	3
p_K	.2	.2	.178

32) For a premium computed using the equivalence principle,

$$P = \frac{APV\ (\text{benefits})}{APV\ (\text{annuity due of 1 per year})}$$

since P APV(annuity due of 1 per year) = APV(benefits)

ex}



For last ex, premiums are paid if the person is active

$t =$	0	1	2	3
$1v^t = v^t$	v^0	v	v^2	v^3
prob	1	0.7	0.59	0.513

So denominator $= 1 + 0.7(1.05)^{-1} + 0.59(1.05)^{-2} + 0.513(1.05)^{-}$

$= 2.64496$

83.5

So the Premium for the disability "insurance"

$$P = \frac{10512.9}{2.64496} = \boxed{3974.69}$$

33) The terminal policy value ~~reserve~~ at time k is ${}_kV$ = APV (future benefits) − APV(future Premiums)

and depends on the state at time k,

So might find the terminal policy value ~~reserve~~ at time 1 if the person is active then.

end old M402 need new material

1] p453 The expected profit in the $(t+1)$st contract year, measured at the end of the contract year, is Pr_{t+1}.

Assuming 2 decrements, $Pr_{t+1} =$

$$\left[{}_tV^G + G_{t+1}(1-r_{t+1}) - e_{t+1}\right](1+i_{t+1}) -$$

$$\left[\left(b^{(1)}_{t+1} + s^{(1)}_{t+1}\right) q^{(1)}_{x+t} + \left(b^{(2)}_{t+1} + s^{(2)}_{t+1}\right) q^{(2)}_{x+t} + {}_{t+1}V^G\, p^{(\tau)}_{x+t}\right],$$

subscript t is for the tth year, $t+1$ for the $(t+1)$st year.

Here an expense $s^{(i)}_{t+1}$ is used to settle a benefit claim due to cause i in the $(t+1)$st year. ${}_tV^G$ is a policy value using gross premiums,

G_t is the gross premium

r_t is the percent of premium expense factor

e_t is the fixed expense.

2] If death is the only decrement, $Pr_{t+1} =$

$$\left[{}_tV^G + G_{t+1}(1-r_{t+1}) - e_{t+1}\right](1+i_{t+1}) -$$

$$\left[(b_{t+1} + s_{t+1})\, q_{x+t} + {}_{t+1}V^G\, p_{x+t}\right]$$

3] $Pr_t \overset{\text{set}}{=} 0$ was used to calculate $G_t \equiv G$, where the quantities were assumed known. Now the quantities are the ones actually observed.

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85

4) $Pr_{t+1} > 0 \Rightarrow$ a gain (profit) in $(t+1)$st year

$Pr_{t+1} < 0 \Rightarrow$ loss in $(t+1)$st year

5) Let q_{x+t} be the survival model.
E_0 = initial expenses incurred at time 0 and from $t=0$ to $t=1$.
E_t = renewal expenses incurred at the start of the year from $t-1$ to t for $t \geq 2$ (take $E_1 = 0$ since E_0 includes expenses from $t=0$ to $t=1$).
Let B = face amount for insurance, eg 100000.

ex) 10 year term insurance

per policy inforce at the start of year t

t	Premium at $t-1$	expenses E_t	interest	Expected death claim	Surplus emerging at t
0		700	0		-700
1	1500	0	82.5	1000	582.5
2	1500	52.5	79.61	1100	427.11
3	1500	"	"	1200	327.11
4	1500	"	"	1300	227.11
5	1500	"	"	1400	127.11
6	1500	"	"	1500	27.11
7	1500			1600	-72.89
8	1500			1700	-172.89
9	1500			1800	-272.89
10	1500	52.5	79.61	1900	-372.89

$1500 = P$

$1500 - 52.5 + 79.61 - 1100 = 427.11$ etc

initial expense $= 400 + 0.2P = 700$

$E_t = 0.035P = 79.61$ $\quad i = .055$

Interest $= (P - E_t)\ .055$

Net cash flows are computed assuming the policy is still in force at the start of the year.

$$q_{60+t} = 0.01 + .001\,t \qquad 0 \le t \le 10.$$

expected death claim $= B\, q_{60+t-1}$

$= 100000\ q_{60+t-1} \qquad t = 1, \dots, 9$

6] The expected cashflow is the last column = surplus. Level premiums

(1500) are more than enough to pay renewal expenses and expected death claims in early years, but with increasing prob of death, not sufficient in later years. The negative cash flow at $t=0$ does not require a reserve since it will have been paid as soon as the policy was issued (the company knows what this expense is).

ex] continued

The emerging surplus is per policy in force at the start of the year.

Assume ${}_tV$ is given for $t = 0, \dots, 9$.

per policy in force at the start of year t

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t	$_{t-1}V$	Premium at $t-1$	E_t	I_t	expected death claim $B\,q_{60+t-1}$	$(_tV)(p_{60+t-1})$	Pr_t
0			700				
1	0	1500	0	82.5	1000	405.95	−700
2	410.05		52.5	102.17	1100	732.73	176.55
3	740.88	"	"	120.36	1200	977.04	126.99
4	988.90	"	"	134.00	1300	1135.15	131.70
5	1150.10	"	"	142.87	1400	1202.86	135.26
6	1219.94			147.71	1500	1175.47	137.61
7	1193.37			145.25	1600	1047.70	138.68
8	1064.74			138.17	1700	813.69	138.41
9	827.76			125.14	1800	466.89	136.72
10	475.45	1500	52.5	105.76	1900	0.00	133.52
							128.71

The reserves are amounts the insurer needs to assign from its assets to support the policy. The reserve $_1V = 410.05$ is required for every policy still in force at time 1. Hence the cost to the insurer is

Dickson p357

$$(_1V)(p_{60}) = (_1V)(1-q_{60}) = 410.05(1-.01) = 405.95$$

So the cost at the end of the year (time t) from $t-1$ to t is $(_tV)(p_{60+t-1})$.

I_t is the interest earned from $t-1$ to t

$$I_t = (_{t-1}V + P - E_t)\underbrace{.055}_{i}$$

Then $Pr_t = ({}_{t-1}V + P - E_t)(1+i) - B\, q_{60+t-1} - ({}_tV)(p_{60+t-1})$

$= (P - E_t)(1+i) + \Delta_t V - B\, q_{60+t-1}$

where $\Delta_t V$ is the change in reserve:

$\Delta_t V = ({}_{t-1}V)(1+i) - ({}_tV)(p_{60+t-1})$.

7) * The vector $\underset{\sim}{Pr} = (Pr_0, \ldots, Pr_n)$ (eg $n=10$)

is the profit vector for the contract.

$(Pr_t)({}_{t-1}p_x)$ = expected profit at the end of year $t-1$ to t.

Let $\Pi_0 = Pr_0$, $\Pi_1 = Pr_1$, $\Pi_t = {}_{t-1}p_x\, Pr_t$ (eg $x=60$)
for $t = 1, 2, \ldots, n$.

The vector $\underset{\sim}{\Pi} = (\Pi_0\, \Pi_1, \ldots, \Pi_n)$

$= (Pr_0,\ Pr_1,\ ({}_1p_x)Pr_2,\ ({}_2p_x)Pr_3, \ldots, ({}_{n-1}p_x)Pr_n)$ ($n-1 = 9$, $n = 10$)

is the profit signature for the contract.

8) * In general, $\Pi_0 = Pr_0$, $\Pi_1 = Pr_1$,
$\Pi_{t+1} = Pr_{t+1}\,({}_tp_x)$. $\underset{\sim}{\Pi} = (\Pi_0, \Pi_1, \ldots, \Pi_n)$.
Π_{t+1} = expected profit in $(t+1)$st contract.

year per policy in force at the start of the year.

9]* The rate of interest r 402 79 97.
p456 460 for discounting the Π_t is called the risk discount rate or rate of return on equity (ROE) or hurdle rate. The interest rate used to accumulate beginning of the year values is i. Usually $r > i$ to compensate insurer for risks (such as interest rates).

10]* The NPV of the expected profits

$$\begin{array}{ccccc} \Pi_0 & \Pi_1 & \Pi_2 & \cdots & \Pi_n \\ \hline | & | & | & \cdots & | \\ 0 & 1 & 2 & & n \end{array}$$

is $NPV = \sum_{k=0}^{n} \Pi_k v_r^k$

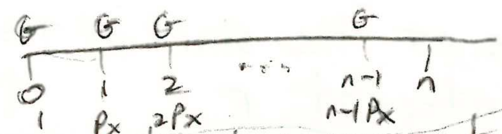
$= \Pi_0 + \Pi_1 (1+r)^{-1} + \dots + \Pi_n (1+r)^{-n}$.

11]* The r_{IRR} that makes $NPV = 0$ is called the internal rate of return. Given r_{IRR}, show $NPV \approx 0$.

12]* The profit margin $= \dfrac{NPV}{APV(\text{premiums})}$

where the APV is computed using the interest rate r used to compute the NPV. For level premiums G, potentially paid at $t = 0, \dots, n-1$, $APV(\text{Premiums}) = G \ddot{a}_{x:\overline{n}|} = G \sum_{k=0}^{n-1} v_r^k \, {}_kp_x$

where ${}_kp_x = \prod_{t=0}^{k-1} (1 - q_{x+t}) = \prod_{t=0}^{k-1} p_{x+t}$.



ex} For the lastex, it can be shown that

$\Pi = (-700, 176.55, 125.72, 128.96, 130.84, 131.39, 130.56, 128.35, 124.76, 119.75, 113.37)$.

Note $q_{60+t} = .01 + .001t$ and

$p_{60+t} = 1 - q_{60+t}$. So $\Pi_2 = \Pi_{1+1}$

$= Pr_{1+1}\,({}_1p_{60}) = Pr_2\,\underbrace{({}_1p_{60})}_{p_{60} = 1 - q_{60}} = 126.99(.99) =$

125.72.

$\Pi_3 = \Pi_{2+1} = Pr_{2+1}\,({}_2p_{60}) = Pr_3\,({}_2p_{60})$

$= 131.7\,(1 - q_{60})(1 - q_{61}) =$

$131.7\,(1 - .01)(1 - .011) = 128.95 \simeq 128.96$

$\Pi_4 = Pr_4\,(1 - .01)(1 - .011)(1 - .012) = 130.85 \simeq 130.84$

using ${}_kp_x = \prod_{t=0}^{k-1} (1 - q_{x+t})$.

13} * The discounted payback period (DPP) (or break even period) is calculated using r, and is the smallest value of m such that $\sum_{k=0}^{m} \Pi_k v_r^k \geq 0$.

The DPP represents the time until the

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88.

insurer starts to make a profit on the contract.

ex] For the running ex, $m = 8$ if $r = 0.1$.

t	$\Pi_t/(1+r)^t = \Pi_t v_r^t$	$NPV(t) = \sum_{k=0}^{t} \Pi_k v_r^k$
$t=0$	$-700/1$	-700
1	$176.55/1.1$	-539.5000
2	$125.72/(1.1)^2$	-435.5991
3	$128.96/(1.1)^3$	-338.7096
4	$130.84/(1.1)^4$	-249.3441
5	$131.39/(1.1)^5$	-167.7613
6	$130.56/(1.1)^6$	-94.0636
7	$128.35/(1.1)^7$	-28.1997
8	$124.76/(1.1)^8$	30.0017
9	$119.75/(1.1)^9$	80.7874
10	$113.37/(1.1)^{10}$	$124.4964 = NPV$

$DPP = 8$

p457

14] $NPV(t)$ = partial net present value

$= \sum_{k=0}^{t} \Pi_k v_r^k$ for $t = 0, 1, \ldots, n-1$ where

$NPV(n) = NPV$.

15] DPP does not exist if $NPV(t) < 0$ for $t = 0, \ldots, n$.

16] Suppose, for example, $r = 0.1$ and $NPV < 0$. Then for the given

88.5

Premium, reserve requirements, and interest, the mortality and expense assumptions do not meet the insurer's $r = 10\%$ profitability standard.

One solution would be to increase the premium if it is believed that the increased premium will be acceptable in the marketplace.

Go to ex on notes 89

§17.2 Uses of Profit analysis

17) Instead of calculating a premium based on the equivalence principle (or percentile method), select the premium to produce a specified r_{IRR} or profit margin (eg 5%).

Skim the rest of §17.2

§17.3 Using Profit Analysis to Determine Reserves

18) Insurance regulators establish a legal minimum for reserves.

19) Consider a profit analysis table with all reserves $= 0$. Then recursively compute reserves with ${}_nV^6 = 0$.

See back of #89

Know for final, Q11, Hw11 100 or 11 exam 3?

402 or 39.

ex)

t	Π_t	v_r^t	$NPV(t) = \sum_{k=0}^{t} \Pi_k v_r^k$ $= NPV(t-1) + \Pi_t v_r^t$	$_tp_x$
0	-5000	$(1) = (1.1)^{-0}$	-5000	1
1	2688.1	$(1.1)^{-1}$	-2556.2727	.985
2	1840.57	$(1.1)^{-2}$	-1035.1405	.96826
3	470.19	$(1.1)^{-3}$	-681.8798	.94986
4	507.8	$(1.1)^{-4}$	-335.0456	.92991
$n=$ 5	363.22	$(1.1)^{-5}$	-109.5145	.90759

$r = 0.1$

2nd column

a) Fill out $NPV(t)$, find DPP if it exists and NPV.

DPP does not exist since $NPV(t) < 0$ for $t = 0, \ldots, 5 = n$.

$NPV = -109.5145$

b) Find APV (premiums) if $G = 19250$

$$APV(\text{Premiums}) = G \sum_{t=0}^{n-1} \frac{_tp_x}{(1+r)^t} = G \sum_{t=0}^{n-1} {_tp_x}\, v_r^t$$

$$= 19250\left(1 + \frac{.985}{1.1} + \frac{.96826}{(1.1)^2} + \frac{.94986}{(1.1)^3} + \frac{.92991}{(1.1)^4}\right)$$

$$= 19250(4.04445) = 77855.7461$$

c) Find profit margin $= \dfrac{NPV}{APV(\text{Premiums})} =$

$$\frac{-109.5145}{77855.7461} = \boxed{-0.001407}$$

d) Show $r_{IRR} \approx .087$. NPV with $r = .087 \approx -5000 + \frac{2688.1}{1.087} + \frac{1840.57}{(1.087)^2} + \frac{470.19}{(1.087)^3} + \frac{507.8}{(1.087)^4} + \frac{363.22}{(1.087)^5} = -0.1558 \approx 0.$

19) continued i) compute the reserves so that the profit vector elements $= 0$,

ii) except if the reserve < 0, set the reserve to 0 and then compute the profit vector element.

This technique is called zeroized reserves.

ex)

p_{x+t-1}	year	pme_t prior reserve + premium − expenses	end of year accumulation $i = .06$	$Pr_0 = -5000$ − expected death benefit $= edb_t$	$=$ Pr_t Year end expected Profit
.985	1	19039	20181.34	15000	5181.34
.983	2	19039	20181.34	17000	3181.34
.981	3	19039	20181.34	19000	1181.34
.979	4	19039	20181.34	21000	−818.66
.976	5	19039	20181.34	$\underbrace{24000}_{edb_t}$	−3818.66

Then $\Pi_5 = Pr_5\,({}_4p_x) = -3818.66(.9299) = -3551.01$

where ${}_tp_x$ is computed as in running ex.

$\Pi_5 = \Pi_n < 0$ is of concern to both the regulators and the insurer.

$$\begin{array}{cccccc} {}_0V^z & {}_1V^z & {}_2V^z & {}_3V^z & {}_4V^z & \\ 0 & 1 & 2 & 3 & 4 & 5 \end{array}$$

recursion! ${}_5V^G = 0 = {}_5V^z$

$$Pr_5 = ({}_4V^z - 19039.0)\,1.06 - 24000 - \underbrace{{}_5V^z}_{0} p_{x+5-1} \overset{\text{set}}{=} 0$$

$${}_4V^z = \frac{24000}{1.06} - 19039.0 = 3602.51$$

402 82
90.

(*) $Pr_t = ({}_{t-1}V^z + pme_t)(1+i) - edb_t - {}_tV^z \, p_{x+t-1}$

$Pr_4 = ({}_3V^z + 19039.0)1.06 - 21000 - 3602.51(.979) = 0$

$\Rightarrow {}_3V^z = 4099.54$

$Pr_3 = ({}_2V^z + 19039)1.06 - 19000 - 4099.54(.981) = 0$

$\Rightarrow {}_2V^z = 2679.54$

$Pr_2 = ({}_1V^z + 19039)1.06 - 17000 - 2679.54(.983)$
$= 0 \quad \Rightarrow {}_1V^z < 0$, so set ${}_1V^z = 0$.

$Pr_1 = ({}_0V^z + 19039)1.06 - 15000 - 0(.985) = 0$

$\Rightarrow {}_0V^z < 0$, so ${}_0V^z = 0$.

Recompute Pr_1 and Pr_2. using (*).

$Pr_1 = (0 + 19039)(1.06) - 15000 - 0(.985) = 5181.34$

$Pr_2 = (0 + 19039)(1.06) - 17000 - 2679.54(.983)$

$= 547.35$

so $\underset{\sim}{Pr} = (-5000, 5181.34, 547.35, 0, 0, 0)$

§17.4 profit distribution

20] Some insurance contracts return part of the profit to the policyholders as a policyholder dividend.

21] For a life insurance with benefit B and only percent of premium expenses, expected profit

$$Pr_{t+1} = \left[{}_tV^G + G_{t+1}(1-r_{t+1})\right](1+i_t) - Bq_{x+t} - {}_{t+1}V^G p_{x+t}.$$

Let primed symbols be the actual observed quantities. Then

$$Pr'_{t+1} = \left[{}_tV^G + G_{t+1}(1-r'_{t+1})\right](1+i'_{t+1}) - Bq'_{x+t} - {}_{t+1}V^G p'_{x+t}.$$

The gross premium, gross premium reserves, and face amount of the insurance are fixed quantities. Then the total gain in the $(t+1)$th contract year, if r, G, i are constant, is

$$G^T = Pr'_{t+1} - Pr_{t+1} =$$

$$\underbrace{({}_tV^G+G)(i'-i)}_{\text{gain from interest}} + \underbrace{(B-{}_{t+1}V^G)(q_{x+t}-q'_{x+t})}_{\text{gain from mortality}} + G\left[r(1+i) - r'(1+i')\right]$$

gain from interest ; gain from mortality

>0 if $i'>i$; >0 if $q_{x+t} > q'_{x+t}$

22] Typically 10-25% of $G^T > 0$ is used for the return.