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$ 17.3

23) The profit distribution can be in cash (a dividend or refund), premium reduction, or the purchase of additional insurance.

ch14) ch16 Universal Life (UL) Insurance

1] UL insurance was first offerred in the 1980's, and combines insurance protection with investment growth.

2] Type A universal life insurance has a death benefit B of a fixed specified amount.
Type B universal life insurance has death benefit = B + account value at the end of the time interval of death.

ex] MLC 301: For a universal life insurance policy on (70):

i) the death benefit at the end of year 10 is the greater of 100000 and 1.3 times the account value at the end of year 10.

ii) $q^{(d)} = 0.01$, $q^{(w)} = 0.03$

iii) The withdrawal benefit is the account value less a surrender charge of 1000.

iv) A premium of 9000 and expenses of 900 were paid at the beginning of year 10.

v) $i = 0.08$ for year 10

vi) The account value at the end of year 10 is 85000.

vii) ${}_9AS = 75000$

Find ${}_{10}AS =$

$$\frac{({}_9AS + G(1-c_9) - e_9)(1+i) - b_{10}\, q^{(1)}_{79} - {}_{10}CV\, q^{(2)}_{79}}{1 - q^{(1)}_{79} - q^{(2)}_{79}}$$

$e_9 = 0$ = per policy expense

$c_9 = \frac{900}{9000} = 0.1$ = fraction of premium for expenses

$b_{10} = \max[100000, 1.3(85000)] = 110500$

withdrawal benefit = ${}_{10}CV = 85000 - 1000 = 84000$. So

$${}_{10}AS = \frac{[75000 + 9000(1-.1))1.08 - 110500(.01) - 84000(.03)}{1 - .01 - .03}$$

$$= \frac{86123.00}{0.96} = 89711.4583$$

3) Let i^c be the interest rate to accumulate the account value. Let i^q be the interest rate

used in the COI calculation. 402 84 92

often $i^c = i^g \equiv i$. (see 43.)

4] The <u>cost of insurance</u> for type B UL is $COI_t = \dfrac{B\, q_{x+t-1}}{1+i^g}$, the

account value

$$AV_t = \left[AV_{t-1} + G_t(1-r_t) - e_t - COI_t\right](1+i^c)$$

and the total death benefit is $B + AV_t$ if the death is in the t th year.

$$AV_0 = 0$$

$$COI_1 = \frac{B\, q_x}{1+i^g}$$

$$AV_1 = \left(0 + G_1(1-r_1) - e_1 - COI_1\right)(1+i^c).$$

If death does not occur in the 1st year,

$$AV_2 = \left(AV_1 + G_2(1-r_2) - e_2 - COI_2\right)(1+i^c)$$

and the recursion continues.

Here r_t = percent of premium expense factor and e_t = fixed expense amount.

5) For type A UL,

$$COI_t = \frac{(B - AV_t)\, q_{x+t-1}}{1+i^q}$$

$$AV_t = \left[AV_{t-1} + G_t(1-r_t) - e_t - \frac{(B-AV_t)\, q_{x+t-1}}{1+i^q}\right](1+i^c)$$

Here AV_t occurs twice. It can be shown that

$$AV_t = \frac{\left[AV_{t-1} + G_t(1-r_t) - e_t - \frac{B\, q_{x+t-1}}{1+i^q}\right](1+i^c)}{(1-q_{x+t-1})\underbrace{\frac{1+i^c}{1+i^q}}_{1 \text{ if } i^c = i^q}}$$

If $i^q = i^c = i$, then

$$AV_t = \frac{\left[AV_{t-1} + G_t(1-r_t) - e_t\right](1+i) - B\, q_{x+t-1}}{p_{x+t-1}}$$

ex] Type B UL insurance $B = 100000$
$r_1 = .75$, $r_t = .1$ if $t > 1$,
$e_1 = 100$, $e_t = 20$ if $t > 1$,
$G_t \equiv 5000$, $i^q = i^c = i = 0.03$

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t	q_{x+t-1}	$AV_{t-1}+G_t(1-r_t)-e_t$	$COI_t = \frac{100000\, q_{x+t-1}}{1.03}$	
0				$AV_0=0$
1	.00076	$0+5000(.25)-100=1150.00$	73.79	1108.50
2	.00081	$1108.5+5000(.9)-20=5588.50$	78.64	5675.16
3	.00085	$5675.16+5000(.9)-20=10155.16$	82.42	10374.82
4	.00090	$10374.82+5000(.9)-20=14854.82$	87.38	15210.46
5	.00095	$15210.46+5000(.9)-20=19690.46$	92.23	20186.18

$AV_t = $ (differene of these 2 terms) 1.03

ex] repeat above ex with typeA UL

$$AV_t = \frac{[AV_{t-1}+G_t(1-r_t)-e_t](1+i) - B\, q_{x+t-1}}{p_{x+t-1}}$$

$$p_{x+t-1} = 1-q_{x+t-1}$$

$$AV_1 = \frac{[0+5000(.25)-100]1.03 - 100000(.00076)}{1-.00076}$$
$$= 1109.34$$

$$AV_2 = \frac{[1109.34+5000(.9)-20]1.03 - 100000(.00081)}{1-.00081}$$
$$= 5680.62$$

$$AV_3 = \frac{[5680.62+5000(.9)-20]1.03 - 100000(.00085)}{1-.00085}$$
$$= 10389.27$$

$$AV_4 = \frac{[10389.27+5000(.9)-20]1.03 - 100000(.00090)}{1-.00090}$$
$$= 15239.06$$

$$AV_5 = \frac{[15239.06 + 5000(.9) - 20]1.03 - 100000(.00095)}{1 - .00095}$$

$$= 20234.86$$

6] If Type A and B UL have the same B, G_t, e_t, r_t, q_{x+t-1}, i^g, and i^c,

then $AV_t(\text{type}A) > AV_t(\text{type}B)$.

since type A gives a smaller death benefit with the same premium.

7] If $i^g = i^c = i$, for type B

$$AV_t = [AV_{t-1} + G_t(1-r_t) - e_t](1+i) - Bq_{x+t-1} = AV_t(B)$$

by 4)

while for type A,

$$AV_t = \frac{[AV_{t-1} + G_t(1-r_t) - e_t](1+i) - Bq_{x+t-1}}{1 - q_{x+t-1}} = AV_t(A)$$

by 5

So if $AV_{t-1}(A) = AV_{t-1}(B)$, then

$$\frac{AV_t(B)}{AV_t(A)} = 1 - q_{x+t-1} < 1.$$

MLC 296 uses this result.

8) p432 At the end of the term of the UL contract, the policyholder receives the final account value. The policyholder can also surrender the UL contract at any time for its cash value (account value − surrender charge).

9) UL insurance as an investment can be taxed at a favorable rate if the death benefit is large enough.

10) UL insurance with a corrider factor has a benefit =

$\min(B, f\,AV_t)$ for type A

$\min(B+AV_t, f\,AV_t)$ for type B

where f is often 2 or 2.5.

Then formulas are hard for calculations.

§16.2

11) Variable Universal life (VUL) insurance has the premiums invested in separate investment accounts, earning interest at market rates. Such a contract transfers the interest rate risk from the insurer to the insured.

12) UL contracts tend to have some premium flexibility: a

minimum amount...... and likely a maximum amount if the insurer guarantees an interest rate, where the insurer could lose money if the market rate falls below the guaranteed rate.

13] UL and VUL contracts with secondary guarantees provide

minimum guaranteed cash values, death benefit, and/or maturity value. (so premiums and reserves are increased.)

14] Equity indexed UL (EIUL)

has interest linked to an average of stock indices eg 100% Dow Jones, or 75% Dow Jones 25% S&P.
(Hard with calculators)

§16.3 15] One method of pricing for secondary guarantees is a shadow fund account based on interest rates higher than the contract's guaranteed min interest and with COI rates lower than the contract's guaranteed max. The shadow account is not available to the policyholder, its purpose is to maintain the death benefit.