402 87

1095

ex)

contract Year	contract account value	surrender charge	cash value	Shadow account value
1	1000	900	100	1100
2	800	800	0	900
3	400	600	0	700
4	100	400	0	500
5	0	200	0	200
6	0	100	0	0

For the above UL contract, when does the contract lapse (end)

a) if there is no secondary guarantee

b) under the shadow fund method for providing a secondary guarantee.

Soln a) the end of the 5th year when the account value becomes 0

b) the end of the 6th year when the shadow account value becomes 0.

§3.4 p.345 §9.4

1] Suppose the UDD assumption (uniform distribution of deaths in 1year interval) holds for all m decrements, then the UDD assumption holds for the total decrement: ${}_tq_x^{(j)} \approx (t)(q_x^{(j)})$ for $j=1,\dots,m$

$\Rightarrow$ ${}_tq_x^{(\tau)} \approx (t)(q_x^{(\tau)})$ for $0 \le t \le 1$.

${}_tp_x^{(k)} \approx 1-(t)(q_x^{(k)})$, $k = j$ or τ.

2] Assume the UDD assumption holds for the jth decrement, then

$${}_tq_x^{(j)} = \int_0^t \underbrace{{}_sp_x^{(\tau)}\mu_{x+s}^{(j)}}_{f_j(s)}\,ds = F_j(t) \overset{\text{by 1)}}{\approx} (t)(q_x^{(j)})$$

So $F_j'(t) \overset{\text{fundamental Th of calc}}{=} \underbrace{{}_tp_x^{(\tau)}\mu_{x+t}^{(j)}}_{f_j(t)} \approx q_x^{(j)}$.

3] Assume the UDD assumption holds for the jth decrement and the total decrement. Let $0 \le t \le 1$. So ${}_tp_x^{(\tau)} \approx 1-(t)\,q_x^{(\tau)}$. (by 1)

Then i) $\mu_{x+t}^{(j)} \overset{\text{by 2)}}{\approx} \dfrac{q_x^{(j)}}{{}_tp_x^{(\tau)}} \approx \dfrac{q_x^{(j)}}{1-(t)(q_x^{(\tau)})}$.

For single decrement probabilities,

ii) $${}_tp_x^{\prime(j)} \approx \left[1-(t)\,q_x^{(\tau)}\right]^{q_x^{(j)}/q_x^{(\tau)}}$$

$$\approx \left[{}_tp_x^{(\tau)}\right]^{q_x^{(j)}/q_x^{(\tau)}}.$$

see HW11 #1

Note that $$\frac{\log\left[{}_tp_x^{\prime(j)}\right]}{\log\left[{}_tp_x^{(\tau)}\right]} \approx \frac{q_x^{(j)}}{q_x^{(\tau)}}.$$

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4] Can also assume that the m single decrement quantities satisfy the UDD assumptions. so

$${}_tq_x^{\prime(j)} \approx (t)\left(q_x^{\prime(j)}\right) \text{ and } {}_tp_x^{\prime(j)}\,\mu_{x+t}^{(j)} \approx q_x^{\prime(j)}$$

for $0 \le t \le 1$ and $j = 1, \ldots, m$.

Then formulas depend on the number m of decrements and on actuarial exams, usually $m=2$ or 3. (97.5)

5] p.247 In 4] assume $m=2$, so $0 \le t \le 1$ and the single decrement quantities satisfy the UDD assumptions.

i) $q_x^{(1)} \approx q_x'^{(1)}\left(1 - \frac{q_x'^{(2)}}{2}\right)$

ii) ${}_tq_x^{(1)} \approx q_x'^{(1)}\left(t - \frac{t^2 q_x'^{(2)}}{2}\right)$, $0 \le t \le 1$

iii) ${}_{t|s}q_x^{(1)} \approx (s)\left(q_x'^{(1)}\right)\left[1 - \left(t + \frac{s}{2}\right) q_x'^{(2)}\right]$,

$0 < s+t \le 1$.

Note i) is ii) with $t=1$.

By symmetry, ${}_tq_x^{(2)} = q_x'^{(2)}\left(t - \frac{t^2 q_x'^{(1)}}{2}\right)$.

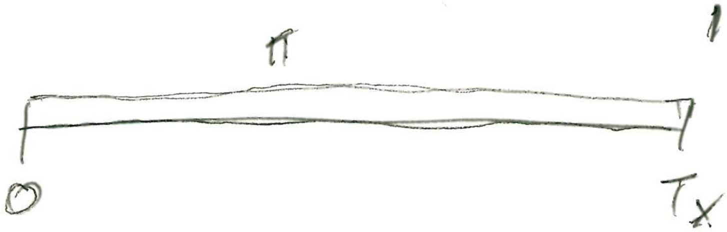
6] p.248 In 4] assume $m=3$. Then

$q_x^{(1)} \approx q_x'^{(1)}\left[1 - \frac{1}{2}\left(q_x'^{(2)} + q_x'^{(3)}\right) + \frac{1}{3} q_x'^{(2)} q_x'^{(3)}\right]$.

end ~~§13.4~~ 9.4 material

see §6.6

1) * Let π be the premium for fully continuous whole life insurance.



$\pi\, a_{\overline{T_x}|}$ = PV of premiums $= \pi \bar{Y}_x$

RV's

$v^{T_x} = e^{-\delta T_x} = \bar{Z}_x$ = PV of unit insurance.

The <u>100 αth percentile premium</u> π is the premium which results in a positive loss at issue with probability α.

So want premium π such that

$$\alpha = P\left(e^{-\delta T_x} > \pi\, \bar{a}_{\overline{T_x}|}\right) = P\left[e^{-\delta T_x} > \pi\left(\frac{1-e^{-\delta T_x}}{\delta}\right)\right]$$

$$= P\left[\underbrace{\left(1+\frac{\pi}{\delta}\right)}_{\frac{\pi+\delta}{\delta}} e^{-\delta T_x} > \frac{\pi}{\delta}\right] = P\left(\underbrace{e^{-\delta T_x}}_{\bar{Z}_x} > \frac{\pi}{\pi+\delta}\right)$$

$$= P\left[-\delta T_x > \log\left(\frac{\pi}{\pi+\delta}\right)\right] = P\left[T_x < -\frac{1}{\delta}\log\left(\frac{\pi}{\pi+\delta}\right)\right]$$

So $P(T_x < t_\alpha) = P\left(\bar{Z}_x > \frac{\pi}{\pi+\delta}\right) = P(\bar{Z}_x > Z_{1-\alpha})$.

So $t_\alpha = \frac{-1}{\delta} \log\left(\frac{\pi}{\pi+\delta}\right)$ and $Z_{1-\alpha} = \frac{\pi}{\pi+\delta}$.

Now $\bar{Z}_x = g(T_x) = e^{-\delta T_x} = v^{T_x}$ is a decreasing function of T_x. So if $t_{1-\alpha}$ is the $1-\alpha$ percentile of T_x, then $e^{-\delta t_{1-\alpha}}$ is the α percentile of $\bar{Z}_x$

So $Z_{1-\alpha} = e^{-\delta t_\alpha}$.

Thus $e^{-\delta t_\alpha} = \frac{\pi}{\pi+\delta}$ or $\pi e^{-\delta t_\alpha} + \delta e^{-\delta t_\alpha} = \pi$.

$\therefore \pi(1-e^{-\delta t_\alpha}) = \delta e^{-\delta t_\alpha}$ or

$$\boxed{\pi = \frac{\delta e^{-\delta t_\alpha}}{1-e^{-\delta t_\alpha}} \underset{\substack{T_x \sim EXP(\mu) \\ \text{nontrivial}}}{=} \frac{\delta (1-\alpha)^{\delta/\mu}}{1-(1-\alpha)^{\delta/\mu}}}$$

If the benefit is K, $\boxed{\pi = \frac{K \delta e^{-\delta t_\alpha}}{1-e^{-\delta t_\alpha}}}$.

(99.5

$$\alpha = P\left[\bar{K} e^{-\delta T_x} > \pi\left(\frac{1-e^{-\delta T_x}}{\delta}\right)\right] =$$

$$P\left[\underbrace{\left(K+\frac{\pi}{\delta}\right)}_{\frac{K\delta+\pi}{\delta}} e^{-\delta T_x} > \frac{\pi}{\delta}\right] = P\left[e^{-\delta T_x} > \frac{\pi}{\pi+K\delta}\right]$$

$$= P\left[\bar{Z}_x > \frac{\pi}{\pi+K\delta}\right]. \quad \text{So } Z_{1-\alpha} = e^{-\delta t_\alpha} = \frac{\pi}{\pi+K\delta}$$

and $\pi e^{-\delta t_\alpha} + K\delta e^{-\delta t_\alpha} = \pi \Rightarrow$

$$\pi = \frac{K\delta e^{-\delta t_\alpha}}{1+e^{-\delta t_\alpha}}$$

see HW 11

2) a) Let $T_x \sim Exp(\mu)$, Then $\alpha = P(T_x < t_\alpha)$

$$= 1 - e^{-\mu t_\alpha} = F_{T_x}(t_\alpha), \therefore$$

$e^{-\mu t_\alpha} = 1-\alpha$ or $-\mu t_\alpha = \log(1-\alpha)$ or

$\boxed{t_\alpha = -\frac{\log(1-\alpha)}{\mu}}$. Thus $e^{-\delta t_\alpha} = e^{\log(1-\alpha)^{\delta/\mu}} = (1-\alpha)^{\delta/\mu}$.

b) If $T_x \sim U(0,\theta)$, then $\boxed{t_\alpha = \alpha\theta}$, where $\theta = \omega - x$ is common.

ch 11 <u>Pension Mathematics</u> 402 100

1) There are 2 major categories for employer sponsered pension plans.

a) The <u>defined contribution (DC) pension plan</u> specifies how much the employer will contribute, as a percentage of salary into the plan. The employee may also contribute. The contributions are invested, and the accumulated funds are available to the employee when s/he leaves the company. (Like an investment account.) can be converted into an annuity

b) The <u>defined benefit (DB) pension plan</u> specifies a level of benefit, usually related to salary near retirement (eg last or max 5 years) or average salary over career. The employer and possibly employee contributions are accumulated. The pension plan actuary monitors the plan funding on a regular basis to assess whether contributions need to be changed.

2) The State of Illinois has failed to pay its contributions for decades. Social security contributions are not enough (government failures not actuary failures).

ex) a) New York City's Teachers Fund (DB)

annual payment =

$1\frac{2}{3}\%$ ($\underbrace{\text{Final "ave salary"}}_{\text{ave last 5 years salary}}$) (years of credited service)

$1\frac{2}{3}\%$ 80000 (40) = 53333.33

b) Social security is DB

3) The replacement ratio

$$R = \frac{\text{pension income in year after retirement}}{\text{salary in year before retirement}}$$

see HW 11 for a calculation.

4) A salary rate function is $\{\bar{s}_y\}_{y \geq 20}$.

For $y \geq x \geq 20$: $\dfrac{\bar{s}_y}{\bar{s}_x} = \dfrac{\text{annual rate of salary at exact age } y}{\text{annual rate of salary at exact age } x}$

ex) $\bar{s}_y = \{1.04^{y-20}\}_{y \geq 20}$ where

an employee aged 30 currently makes 30000.

i) calculate annual rate of salary at age 30.5,

Soln $\dfrac{\bar{s}_y}{\bar{s}_x} = \dfrac{1.04^{30.5-20}}{1.04^{30-20}}$ $\quad \dfrac{\text{answer}}{30000}$

answer $= (1.04)^{0.5}\, 30000 = 30594.12$

5) The salary scale function $\{s_y\}_{y \geq 20}$

satisfies $\dfrac{s_y}{s_x} = \dfrac{\text{salary received in year age } y \text{ to } y+1}{\text{salary received in year age } x \text{ to } x+1}$

For 4) and 5) assume employee remains employed from age x to $y+1$.

101.5

6) $\bar{s}_y \approx s_{y-\frac{1}{2}}$

7) A DB plan has a lifetime retirement benefit, payable at the plan's normal retirement age (z = NRA), often 65.
The projected annual benefit to begin at NRA = z for an employee hired at age x (now) is $PAB_z = 0.01p\, YOS_z\, FAS_z$

where YOS_z = years of service at z = $z-x$

and FAS_z = "final average salary of past k years" at retirement

and p percent of the employee's projected salary (or FAS_z) is used.

8) For final 3 years of retirement,

take $FAS_z = \frac{1}{3}\left(\frac{s_{z-3}+s_{z-2}+s_{z-1}}{s_x}\right) CAS_x$

where CAS_x = current annual salary at age x.

ex) $s_{20}=1$, $s_x = 1.04^{k-20}$, $k=21,22,\ldots,z-1$