ii) A discrete whole life annuity due pays (x) 1 unit at times $t = 0, 1, 2, \ldots$ as long as (x) survives.

The present value RV $\ddot{Y}_x = \ddot{a}_{\overline{K_x+1}|} = Y_x + 1$.

The APV $\ddot{a}_x = E(\ddot{Y}_x) = a_x + 1$

and $V(\ddot{Y}_x) = V(Y_x)$.

28]* A continuous whole life annuity

"makes a continuous payment at annual rate of 1 unit per year as long as (x) survives."

The present value RV $\bar{Y}_x = \bar{a}_{\overline{T_x}|} = \dfrac{1 - v^{T_x}}{\delta}$

$= \dfrac{1 - \bar{Z}_x}{\delta}$. The APV $\bar{a}_x = E(\bar{Y}_x) =$

$\int_0^\infty e^{-\delta t} S_x(t)\,dt$. $V(\bar{Y}_x) = \dfrac{V(\bar{Z}_x)}{\delta^2} =$

$\dfrac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$.

← if T~EXP(μ)

If $T_x \sim EXP(\mu)$, $E\bar{Y}_x \overset{E}{=} \dfrac{1}{\mu + \delta}$.

Know for Quiz1, HW1

7.5

Let $D > 0$.

29] $\underbrace{\int_0^\infty tDe^{-tD}\,dt}_{EX \text{ where } X \sim EXP(D)} = \int_0^\infty \underbrace{e^{-tD}}_{\text{survival function}}\,dt = \frac{1}{D}$.

$\int_0^n \overbrace{De^{-tD}}^{\text{pdf}}\,dt = 1 - e^{-nD}$, $\int_n^\infty De^{-tD}\,dt = e^{-nD}$

$\int_0^n e^{-tD}\,dt = \frac{1}{D}\left[1 - e^{-nD}\right]$

$\int_n^\infty e^{-tD}\,dt = \frac{1}{D}e^{-nD}$

30] i) The discrete immediate n year temporary annuity pays (x) 1 unit at times $t = 1, 2, \ldots n$ if $K_x \geq n$ and at times $1, 2, \ldots, k-1$ if $1 \leq K_x = k-1 \leq n-1$.

(no payment if $K_x = 0$)

present value RV is $Y_{x:\overline{n}|}$

The APV $a_{x:\overline{n}|} = E(Y_{x:\overline{n}|})$.

(both annuities "pay until death" but due starts 1 year earlier)

ii) SKIP The discrete n year temporary annuity due pays (x) 1 unit at times $0, 1, \ldots, n-1$ if $K_x \geq n$

$0, 1, \ldots, k-1$ if $K_x = k-1 < n$.

The present value RV $\ddot{Y}_{x:\overline{n}|} = Y_{x:\overline{n}|} + 1 - Z^{\ \ 1}_{x:\overline{n}|}$

The APV $\ddot{a}_{x:\overline{n}|} = E[\ddot{Y}_{x:\overline{n}|}) = \dfrac{1 - A_{x:\overline{n}|}}{d}$

$= a_{x:\overline{n}|} + 1 - {}_nE_x$ and

$V[\ddot{Y}_{x:\overline{n}|}) = \dfrac{V(Z_{x:\overline{n}|})}{d^2} = \dfrac{{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2}{d^2}$

31]* A continuous temporary n year annuity "makes a continuous payment at annual rate of 1 unit per year" for n years if $T_x > n$ and for T_x years if $T_x < n$.

The present value RV is $\overline{Y}_{x:\overline{n}|} = \dfrac{1 - \overline{Z}_{x:\overline{n}|}}{\delta}$.

The APV $\overline{a}_{x:\overline{n}|} = E(\overline{Y}_{x:\overline{n}|}) = \int_0^n e^{-\delta t} S_x(t)\,dt$

$V(\overline{Y}_{x:\overline{n}|}) = \dfrac{{}^2\overline{A}_{x:\overline{n}|} - (\overline{A}_{x:\overline{n}|})^2}{\delta^2}$

32] i) A discrete immediate n year deferred whole life annuity makes no payment if $K_x \leq n$. If $K_x = k-1 \geq n+1$, unit payment is made at $t = n+1, n+2, \ldots, K-1$.

The present value RV is ${}_{n|}Y_x = Y_x - Y_{x:\overline{n|}}$.

The APV ${}_{n|}a_x = E({}_{n|}Y_x) = a_x - a_{x:\overline{n|}}$.

ii) skip A discrete n year deferred annuity due makes no payment if $K_x < n$. If $K_x = k-1 \geq n$, then unit payment is made for $t = n, n+1, \ldots, k-1$.

The present value RV is ${}_{n|}\ddot{Y}_x = \ddot{Y}_x - \ddot{Y}_{x:\overline{n|}}$

$= Z_{x:\overline{n|}}^{\;\;1} + {}_{n|}Y_x$. The APV

${}_{n|}\ddot{a}_x = E({}_{n|}\ddot{Y}_x) = v^n \, {}_nP_x + {}_{n|}a_x$.

33] A continuous n year deferred annuity makes no payment if $T_x \leq n$. skip → If $T_x = t > n$, then "continuous payment at annual unit rate" is made from time n to t. The present value RV is ${}_{n|}\bar{Y}_x = \bar{Y}_x - \bar{Y}_{x:\overline{n|}}$. The APV

${}_{n|}\bar{a}_x = E({}_{n|}\bar{Y}_x) = \bar{a}_x - \bar{a}_{x:\overline{n|}} = \int_n^\infty e^{-\delta t} S_x(t)\,dt$

$E[({}_{n|}\bar{Y}_x)^2] = \frac{2}{\delta} v^{2n} \, {}_nP_x \, (\bar{a}_{x+n} - {}^2\bar{a}_{x+n})$ where

$\bar{a}_{x+n} = \int_0^\infty e^{-\delta t} S_{x+n}(t)\,dt$ and ${}^2\bar{a}_{x+n} = \int_0^\infty e^{-2\delta t} S_{x+n}(t)\,dt$.

8 5/4

34) Know

i) a) $\mu_{x+t} \equiv \mu$ or constant force mortality $\Leftrightarrow$ $T_x \sim EXP(\mu)$

b) $T_0 \sim EXP(\mu) \Rightarrow T_x \sim EXP(\mu)$

ii) $T_0 \sim DeMoivre(\omega) \sim U(0,\omega)$

$\Rightarrow T_x \sim U(0, \omega - x)$

iii) $T_0 \sim GD(\alpha, \omega) \Rightarrow T_x \sim GD(\alpha, \omega - x)$

Generalized DeMoivre

end Quiz1 HW1 material

end M401 review

M401 ~~M402 9~~ ~~M522 45~~

M401 52

52) p209 A common life table approximation is $\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}$

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53) The illustrative life table gives x, $\ddot{a}_x$, $1000 A_x$, $1000({}^2A_x)$ for $i = 0.06$ (= annual rate). If problem says data follows illustrative life table, use these values.

54) p196, 210 The continuous annuities are the limiting case of the mthly annuities as $m \to \infty$ and are often good approximations for $m \geq 12$.

So $\bar{a}_x \approx \ddot{a}_x - \frac{1}{2}$

Skip § 6.5, 6.6

Ch10

Ch10 1] Multiple life functions pay benefits until an event (status) effecting 2 or more people occurs.

ex] pay benefits until all of a group of heirs die

§10.1 2] p291 A joint life status is for 2 people. (couple) The event is the 1st death.

3) (xy) gives the ages of the 2 people at time 0.

T_{xy} = time until 1st death $= \min(T_x, T_y)$

4) Let $T_x \perp\!\!\!\perp T_y$ denote T_x and T_y are ind independent

5) The survival function $S_{xy}(t) = {}_tP_{xy} = P(T_{xy} > t)$
$= S_{T_x,T_y}(t,t)$.

If $T_x \perp\!\!\!\perp T_y$, then ${}_tP_{xy} = P(\min(T_x, T_y) > t) =$

$P(T_x > t, T_y > t) = P(T_x > t \cap T_y > t) = ({}_tP_x)({}_tP_y)$.

usually assume $T_{x_1} \perp\!\!\!\perp T_{x_2} \perp\!\!\!\perp \cdots \perp\!\!\!\perp T_{x_K}$ so ${}_tP_{x_1 x_2 \ldots x_K} = \prod_{j=1}^{K} {}_tP_{x_j}$.

6) cdf $F_{xy}(t) = 1 - S_{xy}(t) = P(T_{xy} \le t) = {}_tq_{xy} = 1 - {}_tP_{xy}$.

If $T_x \perp\!\!\!\perp T_y$, ${}_tq_{xy} = 1 - ({}_tP_x)({}_tP_y) = 1 - (1 - {}_tq_x)(1 - {}_tq_y)$

$= P(\text{either or both die by time } t) = P(T_x \le t \cup T_y \le t)$

union, not intersection

$= {}_tq_x + {}_tq_y - ({}_tq_x)({}_tq_y)$.

(general addition rule for 2 ind events). Note ${}_tq_{xy} \ne ({}_tq_x)({}_tq_y)$.

convert q's to p's then back to q's if necessary.

7) $P_{xy} = S_{xy}(1) = {}_1P_{xy} = P(T_{xy} > 1)$

$q_{xy} = 1 - P_{xy} = {}_1q_{xy} = P(T_{xy} \le 1)$

8) p 293 pdf $f_{xy}(t) = \frac{d}{dt} F_{xy}(t) = -\frac{d}{dt} S_{xy}(t)$

product rule for derivatives

If $T_x \perp\!\!\!\perp T_y$, $f_{xy}(t) = -\frac{d}{dt}\left[S_x(t) \quad S_y(t)\right] = f_x(t)\, S_y(t) + S_x(t)\, f_y(t)$

$= \left[({}_tP_x)(\mu_{x+t})\right] {}_tP_y + {}_tP_x \left[({}_tP_y)(\mu_{y+t})\right] = {}_tP_{xy}(\mu_{x+t} + \mu_{y+t})$

9] force of mortality function

$$\mu_{xy}(t) = \frac{f_{xy}(t)}{S_{xy}(t)}$$

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~~40155~~

If $T_x \perp\!\!\!\perp T_y$, $\mu_{xy}(t) = \frac{{}_tp_{xy}\,(\mu_{x+t} + \mu_{y+t})}{{}_tp_{xy}} = \underbrace{\mu_{x+t} + \mu_{y+t}}_{\text{constant if } T_x \sim EXP(\mu_x) \perp\!\!\!\perp T_y \sim EXP(\mu_y)}$

has nice formulas

$\equiv \mu_{x+t:y+t} = \mu_{xy}(t)$

10] p291 The concept of <u>status</u> is used to define survival and failure. For the joint life status, survival of the status means both survive: so the status fails when the 1st death occurs.

11] Insurance is payable when the status fails. An annuity is payable as long as the status survives. Insurance can be restricted so they are payable only if individuals die in a specified order.

12] p294 a) ${}_{n|}q_{xy} = P(n < T_{xy} \le n+1)$

Let $w = x+n$, $z = y+n$

$P_{wz} = P_{x+n:y+n}$

$q_{wz} = q_{x+n:y+n}$

Let $p_{x+n:y+n} = \frac{{}_{n+1}p_{xy}}{{}_np_{xy}}$ and $1 - p_{x+n:y+n} = q_{x+n:y+n}$.

Then ${}_{n|}q_{xy} = {}_np_{xy} - {}_{n+1}p_{xy} \overset{\text{not obvious}}{=} {}_np_{xy}\,(1 - p_{x+n:y+n}) = {}_np_{xy}\,q_{x+n:y+n}$

If $T_x \perp\!\!\!\perp T_y$, ${}_{n|}q_{xy} = ({}_np_x)({}_np_y) - ({}_{n+1}p_x)({}_{n+1}p_y)$.

See notes 3/4=10½ for b), c) ← later

10.5

13] $E[T_{xy}] = \mathring{e}_{xy} = \int_0^\infty t\, f_{xy}(t)\, dt = \int_0^\infty {}_tp_{xy}\, dt$

$E[(T_{xy})^2] = \int_0^\infty t^2 f_{xy}(t)\, dt = 2\int_0^\infty t \underbrace{{}_tp_{xy}}_{S_{xy}(t)} dt$

14] p295-6 The curtate expectation of future lifetime for the joint status is $e_{xy} = \sum_{k=1}^{\infty} {}_kp_{xy} = E[K_{xy}]$ (where RV K_{xy} = curtate duration at failure RV for joint status (xy))

The temporary curtate lifetime $e_{xy:\overline{n|}} = \sum_{k=1}^{n} {}_kp_{xy}$ = average # of whole years of survival within the next n years of the joint status (xy).

§12.2 15] * A two life last survivor status for $(\overline{xy})$ has $T_{\overline{xy}} = \max(T_x, T_y)$. So $T_{xy} + T_{\overline{xy}} = T_x + T_y$.

16] cdf $F_{\overline{xy}}(t) = P(T_{\overline{xy}} \le t) = {}_tq_{\overline{xy}} = F_{T_x,T_y}(t,t)$. If $T_x \perp\!\!\!\perp T_y$, $F_{\overline{xy}}(t) = P(\max(T_x, T_y) \le t) = P(T_x \le t \cap T_y \le t)$

$= F_x(t)\, F_y(t)$. So ${}_tq_{\overline{xy}} = ({}_tq_x)({}_tq_y)$ if $T_x \perp\!\!\!\perp T_y$

convert p's to q's then back to p's if needed.

17] Survival function $S_{\overline{xy}}(t) = P(T_{\overline{xy}} > t) = 1 - F_{\overline{xy}}(t) = {}_tp_{\overline{xy}}$.

If $T_x \perp\!\!\!\perp T_y$, $S_{\overline{xy}}(t) = 1 - ({}_tq_x\, {}_tq_y) = 1 - (1 - {}_tp_x)(1 - {}_tp_y)$

$= \underbrace{{}_tp_x + {}_tp_y - ({}_tp_x)({}_tp_y)}_{\text{can be found from a (single) life table}} = {}_tp_x + {}_tp_y - {}_tp_{xy} = {}_tp_{\overline{xy}}$.

$\boxed{\text{see HW2 \#1}}$