Note that individuals who die or are censored at time t_i are "at risk at t_i". So $r_1 = n$ if $t_1 = t_{(1)}$. In interval $[t_{i-1}, t_i]$, the conditional prob of dying $= \frac{d_i}{r_i}$

estimated

and the conditional prob of surviving $= 1 - \frac{d_i}{r_i}$.

14) Know for final) Let $t_0 = 0$. The Kaplan Meier estimator of $S(t_i) = P(Y > t_i)$ is $\hat{S}_K(0) = 1$ and

$$\hat{S}_K(t_i) = \prod_{j=1}^{i} \left(1 - \frac{d_j}{r_j}\right) = \hat{S}_K(t_{i-1})\left(1 - \frac{d_i}{r_i}\right).$$

$\hat{S}_K(t)$ is a step function:

$\hat{S}_K(t) = \hat{S}_K(t_{i-1})$ for $t_{i-1} \le t < t_i$.

If $t_{(n)}$ is uncensored, $\hat{S}_K(t) = 0$ for $t > t_{(n)}$.

If $t_{(n)}$ is censored, $\hat{S}_K(t) = \hat{S}_K(t_m)$ for $t_m \le t < t_{(n)}$

but $\hat{S}_K$ is undefined for $t > t_{(n)}$.

15) Know for final Given, eg,

$T_1, T_2, T_3^*, \ldots, T_{n-1}^*, T_n$

make a table with headers

$t_{(i)}$ t_i r_i d_i $\hat{S}_K(t_i)$

and compute the Kaplan Meier estimator.

convention: if a death and censored observation are tied, data will have censored obs after death,

eg $10, 10^*$.

$r_i = \# \, t_{(j)} \geq t_i$. Let $t_i^* = t_i +$.

ex] 13 survival times See HW6 #1

$36, \underbrace{38, 38, 38+}_{\text{ties}}, 78, 112, 112, 114+, 162+,$
$189, 198, 237, 489+$

step i) write data down in column 1:

could omit

δ_j	$t_{(j)}$ (col 1)	t_i	r_i	d_i	$\hat{S}_K(t_i) = \hat{S}_K(t_{i-1})\left(1-\frac{d_i}{r_i}\right)$
		$t_0 = 0$			$\hat{S}(0) = 1$
1	36	t_1 36	13	1	$\hat{S}(36) = 1\left(1-\frac{1}{13}\right) \approx .9231$
1	38	t_2 38	12	2	$\hat{S}(38) = .9231\left(1-\frac{2}{12}\right) \approx .7692$
1	38		(2 died, 1 censored)		
0	38+				
1	78	t_3 78	9	1	$\hat{S}(78) = .7692\left(1-\frac{1}{9}\right) = .6837$
1	112	t_4 112	8	2	$\hat{S}(112) = .6837\left(1-\frac{2}{8}\right) = .5128$
1	112		(8 left at risk)		
0	114+				
0	162+				
1	189	t_5 189	4	1	$\hat{S}(189) = .5128\left(1-\frac{1}{4}\right) = .3846$
1	198	t_6 198	3	1	$\hat{S}(198) = .3846\left(1-\frac{1}{3}\right) = .2564$
1	237	t_7 237	2	1	$\hat{S}(237) = .2564\left(1-\frac{1}{2}\right) = .1282$
0	489+				

* variant: given table with $t_{(j)}$, fill in t_i, r_i, d_i, $\hat{S}_K(t_i)$

Note: $\hat{S}(112) = \prod_{j=1}^{4}\left(1-\frac{d_j}{r_j}\right) =$ ← faster than doing the whole table

$\left(1-\frac{1}{13}\right)\left(1-\frac{2}{12}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{2}{8}\right) = .5128.$

(39.5)

16} Know The risk set at time t_j:

$= r_j =$ # of observed lives (at risk) to have survived to t_j^- ($t < t_j^-$). Let $c_j =$ # exits (right censored) − # new entrants (left truncated) between times $[t_j, t_{j+1})$. (Book uses $t_{(j)}$ for t_j.)

Suppose r_0 is given. Then

$r_1 = r_0 - c_0$ and

$r_{j+1} = r_j - d_j - c_j$.

$$\hat{S}_K(t_i) = \prod_{j=1}^{i}\left(1-\frac{d_j}{r_j}\right) = \hat{S}_K(t_{i-1})\left(1-\frac{d_i}{r_i}\right)$$

17}* Greenwood's formula

$$\hat{V}\left[\hat{S}_K(t)\right] \approx \left[\hat{S}_K(t)\right]^2 \underbrace{\sum_{j:\, t_j \le t}}_{\text{don't use to}} \frac{d_j}{r_j(r_j-d_j)}$$

$$SE(\hat{S}_K(t)) = \sqrt{\hat{V}(\hat{S}_K(t))}$$

ex)

j	t_j (given)	d_j (given)	c_j (given)	r_j (given)	$\hat{S}(t)$
0	0		8	100	
1	23.5	1	−3	92	$\hat{S}(23.5) = 1\left(1-\frac{1}{92}\right) = .9891$
2	44.2	1	2	94	$\hat{S}(44.2) = .9891\left(1-\frac{1}{94}\right) = .9786$
3	57.0	2	−5	91	$\hat{S}(57.0) = .9786\left(1-\frac{2}{91}\right) = .9571$
4	59.0	1	4	94	$\hat{S}(59.0) = .9571\left(1-\frac{1}{94}\right) = .9469$

Know

Now $\hat{S}(t_i) = \prod_{j=1}^{i}\left(1-\frac{d_j}{r_j}\right) = \hat{S}(t_{i-1})\left(1-\frac{d_i}{r_i}\right)$

$r_0 = 100$

$r_1 = 100 - 8 = r_0 - c_0$

$r_2 = 92 - 1 - (-3) = 94 = r_1 - d_1 - c_1$

$r_3 = 94 - 1 - 2 = 91 = r_2 - d_2 - c_2$

$r_4 = 91 - 2 - (-5) = 94 = r_3 - d_3 - c_3$

So $SE\left(\hat{S}_K(44.2)\right) =$

$$\left[\left[S_K(44.2)\right]^2\left\{\frac{1}{92(91)} + \frac{1}{94(93)}\right\}\right]^{\frac{1}{2}}$$

$$= .9786\sqrt{\frac{1}{92(91)} + \frac{1}{94(93)}} = 0.01496$$

and a 95% CI for $S(44.2)$ is

$\hat{S}(44.2) \pm 1.96\ SE[\hat{S}(44.2)]$ 40.5

$= .9786 \pm 1.96\ (.01496) =$

$.9786 \pm .02932$

$= [.9493,\ 1.0079]$

$\uparrow$ 1 is better

17] p669 The Nelson Aalen estimator

of $H(t)$ is $\hat{H}_N(t_i) = \sum_{j=1}^{i} \frac{d_j}{r_j} = \underbrace{\hat{H}_N(t_{i-1})}_{\hat{H}_N(t_0) = \hat{H}_N(0) = 0} + \frac{d_i}{r_i}$

with $\hat{V}(\hat{H}(t_i)) = \sum_{j=1}^{i} \frac{d_j(r_j - d_j)}{r_j^3}$.

$\hat{H}_N(t)$ is a step function so can replace

$\sum_{j=1}^{i}$ by $\sum_{j:\, t_j \le t}$ to get $\hat{H}_N(t)$

and $\hat{V}(\hat{H}_N(t))$.

18] $\hat{S}_N(t) = e^{-\hat{H}_N(t)}$ step function

$\hat{H}_K(t_i) = -\log(\hat{S}_K(t_i))$ with $\hat{H}_K(0) = 0$

step function

ch 6) Annual Premiums

1] How should companies fund insurance and pension plans (ignoring expenses, and profit)?

ex) 1 unit insurance sold to (x) pays (x) 1 unit eventually, but company does not know when.

2] p203 Suppose uncertainty regarding future payment did not exist. So we know that amount X is needed at time k, and we fund this need by accumulating deposits of size P at times $t = 0, 1, \ldots, k-1$.

3] If deposits earn effective interest i per period (year), then $P\,\ddot{a}_{\overline{k|}} = X v^{k}$, so periodic deposit P is found by equating the APV of the funding scheme to the APV of the contingent payment.
Such funding satisfies the equivalence principle.

4] $\ddot{a}^{*}_{\overline{K}|} = v^0 + v^1 + \dots + v^{K-1} = \ddot{a}_{\overline{K}|i}$ when $v = \frac{1}{1+i}$. 402-42) (:

So $\underbrace{P + P v^1 + \dots + P v^{K-1} = X v^K}_{\text{equivalence principle}}$

5] Let L be the loss RV.

often $L = Z - PY$ and $E(L) = 0$

P ↑ constant premium.

Let $V(L) = V[L(1)]$ be for unit payment.

Then for payment X, $V(L(X)) = X^2 V[L(1)]$.

See Hw6 #4.

9.4

6] For continuous payment funding schemes (582 43) 59

402 20

the funding payment is made continuously at rate $\bar{P}$.

Equivalence principle: $\bar{P}\,\bar{a}$ = insurance or annuity APV.

7] Discrete insurance with continuous premium.

i) whole $\quad \bar{P}_x = \dfrac{A_x}{\bar{a}_x}$

ii) n year term $\quad \bar{P}^{1}_{x:\overline{n}|} = \dfrac{A^{1}_{x:\overline{n}|}}{\bar{a}_{x:\overline{n}|}}$

iii) n year pure endowment $\quad \bar{P}_{x:\overline{n}|}^{\;\,1} = \dfrac{A_{x:\overline{n}|}^{\;\,1}}{\bar{a}_{x:\overline{n}|}}$

iv) n year endowment $\quad \bar{P}_{x:\overline{n}|} = \dfrac{A_{x:\overline{n}|}}{\bar{a}_{x:\overline{n}|}}$.

v) n year deferred insurance $\quad \bar{P}({}_{n|}A_x) = \dfrac{{}_{n|}A_x}{\bar{a}_{x:\overline{n}|}}$.

8]* continuous insurance with continuous premium

has equivalence principle $[\bar{P}(\bar{A})]\,[\bar{a}] = \bar{A}$.

whole: $\bar{P}(\bar{A}_x) = \dfrac{\bar{A}_x}{\bar{a}_x} = \dfrac{1-\delta\,\bar{a}_x}{\bar{a}_x} = \dfrac{1}{\bar{a}_x} - \delta$

$\bar{L}(\bar{A}_x) = \bar{Z}_x - [\bar{P}(\bar{A}_x)]\,\bar{Y}_x = \dfrac{\delta \bar{A}_x}{1-\bar{A}_x}$.

$V[\bar{L}(\bar{A}_x)] = \left(1 + \dfrac{[\bar{P}(\bar{A}_x)]}{\delta}\right)^2 \left[{}^2\bar{A}_x - (\bar{A}_x)^2\right]$

$= \left(\dfrac{1}{\delta\,\bar{a}_x}\right)^2 \left[{}^2\bar{A}_x - (\bar{A}_x)^2\right] = \dfrac{{}^2\bar{A}_x - (\bar{A}_x)^2}{(1-\bar{A}_x)^2}$ *

KNOW

ex] 9.8] $T_x \sim EXP(\mu)$

So $\bar{A}_x = \frac{\mu}{\mu+\delta}$, $\bar{a}_x = \frac{1}{\mu+\delta}$ } ← memorize

and $\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} = \frac{\left(\frac{\mu}{\mu+\delta}\right)}{\left(\frac{1}{\mu+\delta}\right)} = \mu$.

${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta}$. So $V[L(\bar{A}_x)] = \frac{\frac{\mu}{\mu+2\delta} - \left(\frac{\mu}{\mu+\delta}\right)^2}{\left[1 - \frac{\mu}{\mu+\delta}\right]^2}$

$= \cdots = \boxed{\frac{\mu}{\mu+2\delta} = {}^2\bar{A}_x}$

9] n year endowment insurance

$\bar{P}(\bar{A}_{x:\overline{n}|}) = \frac{\bar{A}_{x:\overline{n}|}}{\bar{a}_{x:\overline{n}|}} = \frac{1}{\bar{a}_{x:\overline{n}|}} - \delta$

$= \frac{\delta \bar{A}_{x:\overline{n}|}}{1 - \bar{A}_{x:\overline{n}|}}$. If $T_x \sim EXP(\mu)$,

* Then $\bar{P}(\bar{A}_{x:\overline{n}|}) = \mu$.

10] Some other insurance premiums

i) n year term insurance

$\bar{P}(\bar{A}^{1}_{x:\overline{n}|}) = \frac{\bar{A}^{1}_{x:\overline{n}|}}{\bar{a}_{x:\overline{n}|}}$ * $= \mu$ if $T_x \sim EXP(\mu)$

ii) n year deferred insurance

Premiums paid only during the n year deferral period

$$_n\bar{P}(_{n|}\bar{A}_x) = \frac{_{n|}\bar{A}_x}{\bar{a}_{x:\overline{n}|}}$$

type of premium and in numerator.

iii) n-pay whole life insurance

benefit payable at death, premium payable for n years

$$_n\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_{x:\overline{n}|}}$$

iv) n year deferred annuity

$$\bar{P}(_{n|}\bar{a}_x) = \frac{_{n|}\bar{a}_x}{\bar{a}_{x:\overline{n}|}}$$

(since i) has $P = \mu$ if $T_x \sim EXP(\mu)$, ii)-iv) do not have $P = \mu$ if $T_x \sim EXP(\mu)$.)

11] Let $\bar{L} = \bar{L}(\bar{A}_{x:\overline{n}|})$ and $\bar{P} = \bar{P}(\bar{A}_{x:\overline{n}|})$.

Then $V(\bar{L}) = \left(1+\frac{\bar{P}}{\delta}\right)^2 \left[{}^2\bar{A}_{x:\overline{n}|} - (\bar{A}_{x:\overline{n}|})^2\right] =$

$\frac{{}^2\bar{A}_{x:\overline{n}|} - (\bar{A}_{x:\overline{n}|})^2}{(\quad\quad)^2}$, 12] The difference between whole life (44.5

and endowment (compare

8] with 9] and 11])

is whole life drops the $\overline{n|}$.

13] If $T_x \sim EXP(\mu)$, then

$$V[\bar{L}(\bar{A}_x)] = V(\bar{L}(\bar{A}_{x:\overline{n|}}))$$

$$= V(\bar{L}) = \frac{\mu}{\mu+2\delta} = {}^2\bar{A}_x,$$ and the premiums $= \mu$.

14] The notation $_0L = \bar{L} = L$ is sometimes used.

15] If the benefit is X instead of 1,

multiply the variance formulas

in 8] and 11] by X^2, See HW6 #4

16] $V(L) = \dfrac{{}^2A - (A)^2}{(1-A)^2}$ for $L = L_x, L_{x:\overline{n|}}, \bar{L}_x,$

and $\bar{L}_{x:\overline{n|}}$ if the equivalence principle is used.

§9.1 17] discrete whole life insurance model for (x)

makes payment X at time k if $T_x \in (k-1, k]$ so $K_x = k-1$.

Let $\ddot{a}_x = E(\ddot{Y}_x) =$ expected payment of annuity-due (that makes unit payment at $t=0,\dots,K_x$)

$$\begin{array}{ccccc} P_x & P_x & \cdots & P_x & X \\ 0 & 1 & & k-1 & k \end{array}$$

$A_x = E[Z_x] =$ expected payment of life insurance (that makes unit payment at time K_x+1).

under the equivalence principle, $\underbrace{P_x \ddot{a}_x}_{\text{APV of unit payment annuity}} = \underbrace{X \underbrace{A_x}_{\text{APV of unit payment}}}_{\text{APV of contingent payment}}$.

18] premium payment from insured is like an annuity-due payment to the insurance company.

19] $A_x = 1 - d\ddot{a}_x$, so $P_x = \dfrac{A_x}{\ddot{a}_x} = \dfrac{1-d\ddot{a}_x}{\ddot{a}_x} = \underbrace{\dfrac{1}{\ddot{a}_x}}_{\text{only need annuity APV to find } P} - d = \dfrac{dA_x}{1-A_x}$ where $d = \dfrac{i}{1+i} = iv$.

20]* ch 4 and 5 asked what is the lump sum at time 0 the insured should pay for insurance or an annuity. The lump sum was equal to the APV.

In ch 6, the insured pays an annuity of premiums with annual payment P to receive insurance with payment X and we ask what should the premium P be.

Equivalence principle: APV of annuity paid by insured $P\ddot{a} =$ APV of insurance or annuity paid by the insurance company.

21] i) discrete whole life insurance $P_x = \dfrac{A_x}{\ddot{a}_x}$

ii) discrete n year term insurance $P^1_{x:\overline{n}|} = \dfrac{A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$

455

iii) discrete n year pure endowment $P_{x:\overset{1}{n|}} = \dfrac{A_{x:\overset{1}{n|}}}{\ddot{a}_{x:n|}}$

iv) n year endowment insurance $P_{x:n|} = \dfrac{A_{x:n|}}{\ddot{a}_{x:n|}} = \dfrac{1}{\ddot{a}_{x:n|}} - d = \dfrac{d\, A_{x:n|}}{1 - A_{x:n|}}$.

Note the funding notation replaces A by P.

22] $P_{x:n|} = P^{1}_{x:n|} + P_{x:\overset{1}{n|}}$

contract premium covers benefits expenses, and profit. Benefit premium covers benefits.

p205

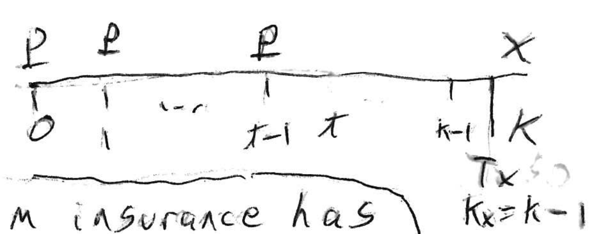
23] Payments P are called annual premium, net annual premiums or benefit annual premiums if P is found using the equivalence principle.

24] Funding should not extend beyond the event that triggers payment nor beyond n for n year term, pure endowment or endowment insurance.

25] Limited payment funding patterns fund the insurance for at most the 1st t years where $t < n$ for n year contracts and $t < \infty$ for whole life contracts. Then P is called a limited payment net premium or a t-pay net premium.

26] i) limited payment (t-pay) whole life insurance has

$${}_tP_x = \frac{A_x}{\ddot{a}_{x:t|}}$$



ii) limited payment (t-pay) n year term insurance has

$${}_tP^{1}_{x:n|} = \frac{A^{1}_{x:n|}}{\ddot{a}_{x:t|}} \qquad t < n$$

M582 57
402 46

27] n year deferred insurance has

$$\underbrace{P({}_{n|}A_x)}_{\text{notation for "}P\text{"}} = \frac{{}_{n|}A_x}{\ddot{a}_x}$$

and

limited payment (t-pay) deferred insurance has

$${}_tP({}_{n|}A_x) = \frac{{}_{n|}A_x}{\ddot{a}_{x:\overline{t}|}} \quad \text{where } t \le n.$$

28] (semi) Continuous insurance pays the claim "immediately." Premiums are still paid by the insured as an annuity-due.

i) whole life $P(\bar{A}_x) = \dfrac{\bar{A}_x}{\ddot{a}_x}$

ii) n year term insurance $P(\bar{A}^{\,1}_{x:\overline{n}|}) = \dfrac{\bar{A}^{\,1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$

iii) n year endowment insurance $P(\bar{A}_{x:\overline{n}|}) = \dfrac{\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$

29] The unit annuity-due expected payments are $\ddot{a}_x$ if $T_x \in (k-1, k]$ so $K_x = k-1$. for whole ins,

$\ddot{a}_{x:\overline{n}|}$ for n year insurance

$\ddot{a}_{x:\overline{t}|}$ for premiums paid at most at times $0, 1, \ldots, t-1$

30] n year deferred annuities with annual premiums P use

$$[P]\underbrace{[\ddot{a}_{x:\overline{n}|}]}_{\text{expected payment by insured}} = \underbrace{{}_{n|}\ddot{a}}_{\text{expected annuity payment by insurance company}}$$

multiply

31] i) n year deferred immediate annuity has 46.5

$$P(n|a_x) = \frac{n|a_x}{\ddot{a}_{x:\overline{n}|}}$$

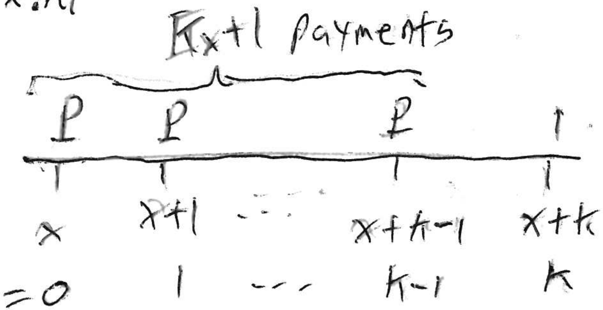
ii) n year deferred annuity due has

$$P(n|\ddot{a}_x) = \frac{n|\ddot{a}_x}{\ddot{a}_{x:\overline{n}|}}$$

iii) n year continuous deferred annuity has

$$P(n|\bar{a}_x) = \frac{n|\bar{a}_x}{\ddot{a}_{x:\overline{n}|}}$$

$K_x = K-1$



32] If (x) fails in kth time interval so unit payment is made at time k, then v^k is the present value of the payment. If the payment is funded by premiums P made at the beginning of each time interval until failure, then $P\ddot{a}_{\overline{K}|}$ is the APV of the funding.

Then $v^k - P\ddot{a}_{\overline{K}|}$ is the present value of the loss. Loss <0 means $P\ddot{a}_{\overline{K}|} > v^k$ and the insurance company makes money.

p210-212 Discrete

582 58
402 247

33] whole life insurance has loss RV $L_x = v^{K_x+1} - P\, \ddot{a}_{\overline{K_x+1|}} = Z_x - P\, \underbrace{\ddot{Y}_x}_{\frac{1-Z_x}{d}}$.

$E[L_x] = A_x - P\, \ddot{a}_x$

$V(L_x) = \left(1 + \frac{P}{d}\right)^2 \left[{}^2A_x - (A_x)^2\right]$.

Under the equivalence principle, P is selected so $E(L_x) = 0$ or $P = \frac{A_x}{\ddot{a}_x} = P_x$.

Then $V(L_x) = \left(\frac{1}{d\ddot{a}_x}\right)^2 \left[{}^2A_x - (A_x)^2\right] = E[(L_x)^2]$

$= \frac{{}^2A_x - A_x^2}{(1-A_x)^2}$.

34] Other insurances funded by premiums are similar. Under the equivalence principle, $E(L) = 0$. So $P = \frac{A}{\ddot{a}}$ and $V(L) = E(L^2)$.

35] In general $L = b_{K_x+1} v^{K_x+1} - P\, \ddot{Y} = Z - P\, \underbrace{\ddot{Y}}_{\text{annuity-due}}$,

The equivalence principle $P = \frac{E[b_{K_x+1} v^{K_x+1}]}{\ddot{a}} = b\frac{E[Z]}{\ddot{a}}$

if $b_{K_x+1} = b$.

36] i) whole life, $L_x = v^{K_x+1} - P_x \ddot{a}_{\overline{K_x+1|}} = Z_x - P_x \ddot{Y}_x$, $P_x = \frac{A_x}{\ddot{a}_x}$.

ii) n year term insurance

$L^1_{x:\overline{n|}} = Z^1_{x:\overline{n|}} - P^1_{x:\overline{n|}} \ddot{Y}_{x:\overline{n|}}$) $\qquad P^1_{x:\overline{n|}} = \frac{A^1_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}}$

iii) n year pure endowment

$$L_{x:\overset{1}{\overline{n}|}} = Z_{x:\overset{1}{\overline{n}|}} - P_{x:\overset{1}{\overline{n}|}}\, \ddot{Y}_{x:\overline{n}|}, \quad P_{x:\overset{1}{\overline{n}|}} = \frac{A_{x:\overset{1}{\overline{n}|}}}{\ddot{a}_{x:\overline{n}|}}$$

iv) n year endowment

$$L_{x:\overline{n}|} = Z_{x:\overline{n}|} - P_{x:\overline{n}|}\, \ddot{Y}_{x:\overline{n}|}, \quad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

with $V(L_{x:\overline{n}|}) = \left(1+\frac{P_{x:\overline{n}|}}{d}\right)^2\left[{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2\right]$

$$= \frac{{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2}{(1-A_{x:\overline{n}|})^2}$$

37) For these 4 models, $L = Z - P\ddot{Y}$

and $\ddot{Y} = \frac{1-Z}{d}$ with $V(L) = \left(1+\frac{P}{d}\right)^2\left[{}^2A - (A)^2\right]$

38) Let $V(L) \equiv V[L(1)]$ be for unit payment. Then for payment x,

$V[L(x)] = x^2\, V[L(1)]$. See 5).

39) i) Deferred annuities immediate: $L = {}_{n|}Y_x - P({}_{n|}a_x)\,\ddot{Y}_{x:\overline{n}|}$, $P({}_{n|}a_x) = \frac{{}_{n|}a_x}{\ddot{a}_{x:\overline{n}|}}$

ii) due: $L = {}_{n|}\ddot{Y}_x - P({}_{n|}\ddot{a}_x)\,\ddot{Y}_{x:\overline{n}|}$, $P({}_{n|}\ddot{a}_x) = \frac{{}_{n|}\ddot{a}_x}{\ddot{a}_{x:\overline{n}|}}$

Note: Sometimes Π is used instead of P for the premium.