Math 404 HW 10 Spring 2024. **One page, 5 problems.** Due Tuesday, April 16. Problem 1 is fair game for the Thursday, April 11 quiz 9. Exam 3 is Thursday, April 25. The final is Monday, May 6, 8-10 AM in the morning in EGRA 322 (usual class room).

1) Suppose $X \sim \text{Poisson}(\lambda)$ where $\lambda \sim G(\alpha = 2, \theta = 1)$. Find Buhlmann's Z if $n = 1$.

2) You are asked to determine the upcoming (2016) premium for company A. The expected aggregate annual loss dollars for risks that are similar to company A is 10,000,000. The number of claims follows a Poisson distribution, and claim severity follows an exponential distribution. You have loss data for company A for one year (2014) when there were 220 claims, resulting in total aggregate losses of 7,350,000. The risk's exposure for 2016 is identical to the exposure during 2014. The full credibility standard is defined as total claim costs being with 4% of the expected aggregate losses 90% of the time. Determine the limited fluctuation credibility premium P_C for 2016 expressed in terms of number of claims.

Hint: $e = n_F$ and $W = S$. $CV(X)$ does not depend on θ when $X \sim EXP(\theta)$, $Z = \sqrt{n/n_F}$ where $n = 220$. $T = S = 7,350,000$. $P_C = M + Z(T - M)$.

3) An insurer has data on losses for five policyholders for ten years. The loss from the ith policyholder for year j is X_{ij}.

$$\sum_{i=1}^{5}\sum_{j=1}^{10}(X_{ij} - \overline{X}_i)^2 = 118.95, \quad \text{and} \quad \sum_{\mathrm{i=1}}^{5}(\overline{\mathrm{X}}_{\mathrm{i}} - \overline{\mathrm{X}})^2 = 4.73.$$

Using nonparametric empirical Bayes estimation, calculate the Buhlmann credibility factor $\hat{Z}$ for an individual policyholder.

4) For a group of 1000 policyholders, the number of claims for each policyholder has a conditional Poisson distribution. In Year 1, the group had the following claims experience: of the 1000 policyholders, 500 had 0 claims, 300 had 1 claim, 150 had 2 claims, and 50 had 3 claims. A randomly selected policyholder from the group had 3 claims in Year 1. Determine E, the semiparametric empirical Bayes estimate of the number of claims in Year 2 for the same randomly selected policyholder.

5) A car insurer entering a new territory assumes each individual policyholder's claim count is $\text{Poisson}(\lambda)$ where the prior $\lambda \sim G(\alpha = 50, \theta = 0.002)$. During the subsequent two year period the insurer covered 750 and 1100 cars in the first and second years with 65 and 112 claims in the first and second years.

a) Find the posterior gamma distribution for λ.

b) Find the coefficient of variation of the distribution in a).

Exam C problems. (classical) credibility: 2, 27, 39, 65, 148, 159, 245, 273, 287
Buhlmann credibility: 8, 18, 32, 35, 41, 48, 62, 67, 70, 78, 133, 151, 154, 177, 181, 187, 190, 200, 215, 219, 230, 236, 251, 267
Semiparametric Empirical Bayes Buhlmann credibility: 197, 240, 257
Nonparametric Empirical Bayes Buhlmann credibility: 12, 38, 145, 194, 223, 270