Math 404 HW 2 Spring 2024. Due Tuesday, Jan. 30. **1 page, 4 questions**

Quiz 2 will cover method of moments estimators.

1) Let X have cdf $F(x) = (1 - \epsilon)F_{X_1}(x) + \epsilon F_{X_2}(x)$ where $0 \leq \epsilon \leq 1$, $P(X_1 = 0) = 1$, and $X_2 \sim \text{Poisson}(\lambda)$.
   a) Find the method of moments estimator of λ if ϵ is known.
   b) Find the method of moments estimators of ϵ and λ if both parameters are unknown.
   c) If there is a sample of size 1000 from a population having the distribution of X with $\epsilon = 0.5$ known, and 570 of the values are 0, estimate λ.
   Hint: Suppose $\epsilon = 0.5$. Then $P(X = 0) = 0.5 + 0.5P(X_2 = 0)$ where $X_2 \sim \text{Poisson}(\lambda)$.

2) A random sample from a negative binomial distribution X results in the following observed number of claims during a given period:

number of claims	number of observations
0	6
1	0
2	4
3+	0

   a) Find $\overline{x} = m$.
   b) Find $\hat{\sigma}_E^2$.
   c) Find the method of moments estimators of r and β using a) and b).
   d) Use the MMEs from c) to estimate the probability of having one or more claims.

3) The three observations 2, 10, and 25 were obtained as a random sample from a gamma distribution with unknown parameters α and θ. (See example done in class.)
   a) Find the method of moments estimators of α and θ.
   b) Use the estimators from a) to determine (estimate) $E(X^3) = \dfrac{\theta^3 \Gamma(\alpha + 3)}{\Gamma(\alpha)}$.

4) Suppose $\sum_{i=1}^{100} X_i = 468$ and $\sum_{i=1}^{100} X_i^2 = 2678$. Then $m = 4.68$ and $t = 26.78$. Using formulas from Exam 1 review 23), find the method of moments estimators for
   a) $V(X)$,
   b) μ and σ^2 if $X \sim LN(\mu, \sigma)$,
   c) λ if $X \sim \text{Poisson}(\lambda)$,
   d) θ if $X \sim U(0, \theta)$,
   e) α and θ if $X \sim G(\alpha, \theta)$.

From the Exam C practice problems, C6, C19, C49, C75, C143, C146, C193, C232, and C239 are method of moments problems.