Math 404 HW 3 Spring 2024. Due Tuesday, Feb. 6. 1 page 5 questions

Quiz 3 will cover method of moments estimators, percentile matching, and MLEs.

1) Suppose losses follow an EXP(θ) distribution. Find θ by matching the 50th percentile using the table below.

interval (a,b]	proportion
(0,277]	0.50
(277,500]	0.21
(500,700]	0.11
(700,800]	0.04
(800,1000]	0.06
(1000,7000]	0.08

2) You are given the following sample data.

1 1 2 2 4 5 8 15

Find the smoothed empirical estimate of the 70th percentile, and use it to determine the percentile matching estimate of the mean θ of an exponential distribution.

3) You are given the following sample data, with nine observations listed from smallest to largest.

4 8 12 18 24 36 48 58 74

You hypothesize that these data came from a two–parameter Pareto distribution, with parameters $\alpha = 2$ and θ. a) Find the smoothed empirical estimate of the 85th percentile and b) use it to determine the percentile matching estimate $\hat{\theta}$.

4) A random variable is suspected of being a single-parameter Pareto distribution with $\theta = 10$ and parameter α.

a) Find the maximum likelihood estimator (MLE) of α.

b) You observe the following sample of values from the distribution. What is the MLE of α?

11 11 12 12 13 14 17 19

5) The observations 2, 3, and 9 were obtained as a random sample from a distribution with pdf $f(x) = \alpha x^{-(\alpha+1)}$ for $x > 1$. a) Write down $L(\alpha)$ if $x_1 = 2$, $x_2 = 3$ and $x_3 = 9$.

b) Use a) to find the MLE of α.

From the Exam C practice problems, C1, C54, and C246 are percentile matching problems while C4, C14, C26, C34, C37, C44, C56, C61, C69, C79, C137, C146, C152, C156, C179, C180, C196, C199, C217, C218, C225, C229, C239, C250, C256, 259, C262, C276, and C277 are MLE problems.