Math 404 HW 4 Spring 2024. Due Tuesday, Feb. 20. Exam 1 Tue. Feb. 13.

1) Problem C4: Losses follow a single parameter Pareto distribution with pdf $f(x) = \frac{\alpha}{x^{\alpha+1}}$ where $x > 1$ and $\alpha > 0$. A random sample of size five produced three losses with values 3,6, and 14, and two losses exceeding 25. Find the MLE of α.

Hint: HW1 explains how to find Exam C problems and solutions. Find θ and $[1 - F(25)]^2$ using page 1 of the Exam 1 review.

2) You believe that a Poisson distribution, with a mean of λ, reflects the number of claims per policy each year. You observe how many claims occur under each of three policies during a year, and compile the following observation data:

Claims per Policy	Number of Observations
0 claims	2
1 or more claims	1

Calculate the maximum likelihood estimate of λ.

Hint: 2 $x_i = 0$ and 1 x_i is censored at 1. Since the distribution is discrete, $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$. (Do not use $1 - F(1) = 1 - p_0 - p_1$ for the censored observation.) So the likelihood $L(\lambda) = p_0^2[1 - p_0] = x^2(1 - x) = g(x)$ where $x = p_0 \in (0, 1)$. Find the x that maximizes $g(x)$ and solve for λ.

3) This problem will demonstrate the difference between finding a maximum likelihood estimate with complete versus incomplete data. Suppose that an insurance policy has a deductible of 200 and a policy limit (maximum insurer payment per loss) of 800. Suppose there were a total of ten losses, with ground up values of:
50 100 100 250 400 500 750 900 1,200 2,500.

a) Despite the fact that the deductible and policy limit could serve to truncate or censor some of the data from the insurers perspective, assume that the above losses are from complete data X: no censoring or truncation. You assume that the ground-up losses were generated by a distribution of the form $f(x) = \lambda e^{-\lambda x}$ for $x > 0$. Find the maximum likelihood estimate of λ. Show your development of the likelihood function.

b) Now suppose that you do NOT have complete ground up loss information. Instead, you only have insurer payment information Y^P, which is censored and truncated at the given deductible and policy limit. Convert the above loss data for X to what you would have based on insurer payments $Y^P = \min((X - d)_+, 800)$ provided $Y^P > 0$. Then determine the maximum likelihood estimate of λ. Show your development of the likelihood function. (Assume that the insurer does not even know about any losses below the deductible. So the converted data set has 7 observations, 2 of which are censored. Note that you have to reconvert the Y^P values to X values that have been censored and truncated.)

4) Find $I_1(\theta)$ for the Weibull(θ, τ) distribution with τ known (a 1PREF). Use $E(X^\tau) = \theta^\tau$. (Derive $I_1(\theta)$. Do not write down the answer from the Exam 2 review.)

TURN OVER FOR TWO MORE PROBLEMS

5) C44: You are given losses follow an exponential distribution with mean θ. Using the table below, find the MLE of θ.

Loss Range	Frequency
[0, 1000]	7
(1000,2000]	6
(2000, oo)	7

Hint write down the likelihood as a function of g(p) where $p = e^{-1000/\theta}$. Then maximize $\ln g(p)$ and solve for $\hat{\theta}$.

6) C56: You are given the following information about a group of policies.

Claim Payment	Policy Limit
5	50
15	50
60	100
100	100
500	500
500	1000

Determine the likelihood function.