

Math 404 HW 6 Spring 2024. Due Tuesday, March 5. 5 problems. Two pages.

1) Suppose the random variable  $X$  has pdf  $\lambda e^{-\lambda x}$  for  $x > 0$ .

a) Derive  $I_1(\lambda)$ .

b) Using a) find the asymptotic variance of the MLE of  $\tau(\lambda) = 1/\lambda$  if  $X_1, \dots, X_n$  are iid with the same distribution as  $X$ .

Hints: a) Use Exam 2 review 62). b) Use Exam 2 review 70).

2) C25: The distribution of accidents for 84 randomly selected policies is as follows.

number of accidents	number of policies
0	32
1	26
2	12
3	7
4	4
5	2
6	1
total	84

Decide whether a Poisson, Negative Binomial or Binomial distribution best fits this data by

a) computing the first 4 values of  $kn_k/n_{k-1}$  and

b) computing  $\bar{X}$  and  $\hat{\sigma}_U^2$ .

3) C13 (modified): A particular line of business has three types of claims. The historical probabilities and the number of claims for each type in the current year are shown below. The null hypothesis is that the probability of each type of claim in the current year is the same as the historical probability. Do the 4 step  $\chi^2$  test.

type	historical probability	number of claims in current year
A	0.2744	112
B	0.3512	180
C	0.3744	138

4) C47 (modified): You are given the following observed frequency data collected over a period of 365 days. Fit a Poisson distribution to the data using the MLE, and perform a 4 step  $\chi^2$  test. Hint: the MLE can be obtained using the left table, but the right table is needed for the  $\chi^2$  test.

# claims per day	observed # of days	for test	# of claims per day	observed # of days
0	50		0	50
1	122		1	122
2	101		2	101
3	92		3+	92
4+	0			

Exam C problems on recent topics: (a,b,0) distribution: 25, 94, 108, 166

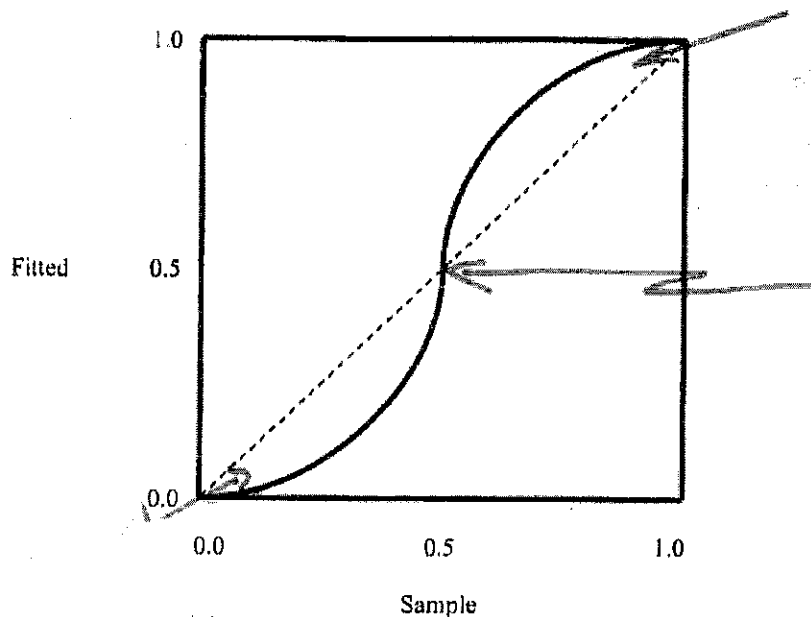
p-p plot: 59, 176, 241

$\chi^2$  test: 13, 23, 47, 71, 140, 149, 189, 201, 222, 244

Kolmogorov Smirnov test: 40, 149, 160, 172, 189, 244, 241

5) Hint: The median is the 50th percentile, and the fitted and sample distributions have the same probability at the median. To tell whether the fitted distribution has more or less probability **around** the median, see whether the slope of the p-p plot curve is greater than or less than one at 0.5.

5) The graph below shows a p-p plot of a fitted distribution compared to a sample.



Which of the following is true?

- (a) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has less probability around the median than the sample.
- (b) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has more probability around the median than the sample.
- (c) The tails of the fitted distribution are too thick on the left and on the right, and the fitted distribution has more probability around the median than the sample.
- (d) The tails of the fitted distribution are too thin on the left and on the right, and the fitted distribution has less probability around the median than the sample.
- (e) The tail of the fitted distribution is too thin on the left and too thick on the right, and the fitted distribution has less probability around the median than the sample.