Math 404 HW 7 Spring 2024. Due Tuesday, March 26. One page, 4 problems.
Exam 2: Thursday March 21, Final: Monday May 6 8-10 AM in the morning

1) You fit various loss models to 20 loss observations using maximum likelihood. The fits maximizing the likelihood for a given number of parameters have the following loglikelihoods.

number of parameters	maximal loglikelihood
1	-142.32
2	-140.75
3	-139.40
4	-138.30
5	-137.40

a) Using the likelihood ratio algorithm (test) at 95% confidence (5% significance), how many parameters are in the selected model? Show how the selected model was chosen.

b) Using the Schwarz Bayesian Criterion, how many parameters are in the selected model?

2) You are given a sample of five claim payments is 110, 120, 150, 200, and 300. Claim sizes are assumed to follow a single parameter Pareto distribution with $\theta = 100$, and $\alpha = 0.4$. Perform the 4 step Kolmogorov Smirnov test, showing the table from exam 3 review 96).

3) See C22. You fit a Pareto distribution to a sample of 200 claim amounts and use the likelihood ratio test to test H_0: data is from the Pareto$(\alpha = 1.5, \theta = 7.8)$ distribution vs H_1: data is from the Pareto$(\hat{\alpha} = 1.4, \hat{\theta} = 7.6)$ distribution. The loglikelihood function evaluated at the MLE is -817.92. and $\sum \ln(x_i + 7.8) = 607.64$. Perform the 4 step LRT.

4) C157: In a portfolio of risks, each policyholder can have at most one claim per year. The probability of a claim for a policyholder during a year is q. The prior density is $\pi(q) = \dfrac{q^3}{0.07}$ where $0.6 < q < 0.8$. A randomly selected policyholder has one claim in year 1 and zero claims in year 2. For this policyholder, determine the posterior probability that $0.7 < q < 0.8$.

Hint: the likelihood function is Bernoulli(q) = binomial$(q, m = 1)$ since there are 0 or 1 claims per year. So the posterior distribution $\pi(q|1,0) \propto f(1|q)f(0|q)q^3 = cq(1-q)q^3$ where the constant c is such that $\int_{0.6}^{0.8} cq(1-q)q^3 dq = 1$. Want $\int_{0.7}^{0.8} cq(1-q)q^3 dq$.

Exam C problems Bayes-Schwarz: 266
LRT: 22, 235