Math 404 HW 8 Spring 2024. Due Tuesday, April 2. One page, 4 problems.

1) Following the example done in class, suppose claim sizes follow a single parameter Pareto distribution with $\alpha = 3$ and Θ where $\Theta \sim U(1,4)$. An insured selected at random submits 4 claims of sizes 2, 3, 5, and 7. Let $f(x_{n+1}|\boldsymbol{x}) = f(x|\boldsymbol{x})$ be the predictive density for $X_{n+1}|\boldsymbol{x}$. It was shown in class that

$$f(x|\boldsymbol{x}) = \frac{39}{(2^{13}-1)16} \, \frac{2^{16}-1}{x^4} \quad \text{for} \quad \text{x} \geq 2, \quad \text{and}$$

$$f(x|\boldsymbol{x}) = \frac{39}{(2^{13}-1)16} \, \frac{x^{16}-1}{x^4} \quad \text{for} \quad 1 \leq \text{x} \leq 2.$$

Find the posterior probability that the next claim will be less than 1.5

$$= P(X_{n+1} \leq 1.5|\boldsymbol{x}) = \frac{39}{(2^{13}-1)16} \int_1^{1.5} x^{12} - x^{-4} \, dx.$$

2) C5: The annual number of claims for a policyholder has a binomial distribution with probability function:

$$p(x|q) = \binom{2}{x} q^x (1-q)^{2-x}$$

for $x = 0, 1, 2$. The prior distribution is $\pi(q) = 4q^3$ where $0 < q < 1$. This policyholder had one claim in each of Years 1 and 2. Determine the Bayesian estimate of the number of claims in Year 3

Hint: $p(x|q)$ is the likelihood when $n = 1$. Recognize that the posterior is a beta distribution. The expected number of claims is $E(X|q) = 2q$, so the Bayesian estimate $E(2q|1,1)$.

3) C11: Losses on a companys insurance policies follow a Pareto distribution with probability density function:

$$f(x|\theta) = \frac{\theta}{(x+\theta)^2}$$

for $0 < x < \infty$. For half of the company's policies $\theta = 1$, while for the other half $\theta = 3$. For a randomly selected policy, losses in Year 1 were 5. Determine the posterior probability that losses for this policy in Year 2 will exceed 8.

Hint: since the prior takes on 2 values, it is a pmf. Use Bayes' theorem for a pmf. Then "$P(X_2 > 8|X_1 = 5)$" $= P(X_2 > 8|\theta = 1)P(\theta = 1|X_1 = 5) + P(X_2 > 8|\theta = 3)P(\theta = 3|X_1 = 5)$ where $P(X_2 > 8|\theta) = 1 - F_W(8)$ if $W \sim Pareto(\alpha = 1, \theta)$.

4) C24: The probability that an insured will have exactly one claim is θ. The prior distribution of θ has pdf

$$\pi(\theta) = \frac{3}{2} \sqrt{\theta}$$

for $0 < \theta < 1$. A randomly chosen insured is observed to have exactly one claim. Determine the posterior probability that θ is greater than 0.6.

Hint: You are given $f(1|\theta) = \theta$ which is what is needed to compute the posterior.

Exam C Bayesian problems: 5, 11, 15, 24, 29, 43, 45, 55, 58, 60, 64, 76, 136, 142, 157, 184, 191, 203, 226, 242, 247, 253, 254, 260,