old E1 MWF too short

Math 404 Exam 1 Spring 2016 Name____________________

YOU ARE BEING GRADED FOR WORK

e 1) You are given the following data:
1, 1, 2, 2, 4, 6, 6, 7, 9, 11, 12, 15, 17. Find the smoothed empirical estimate, $\hat{\pi}$, of the 70th percentile.

(a) $\hat{\pi} \le 8.5$
(b) $8.5 < \hat{\pi} \le 9.0$
(c) $9.0 < \hat{\pi} \le 9.5$
(d) $9.5 < \hat{\pi} \le 10$
(e) $10 < \hat{\pi}$

$n = 13 \quad \lfloor (n+1)p \rfloor = \lfloor 14(.7) \rfloor = \lfloor 9.8 \rfloor = 9 = j$

$h = 9.8 - 9 = 0.8$

$\hat{\pi}_{.7} = (1-h)\, x_{(j)} + h\, x_{(j+1)} =$

$0.2(9) + 0.8(11) = \boxed{10.6}$

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e 2) The observations 5, 10, 25, 50, and 250 were obtained as a random sample from a Gamma distribution with unknown parameters α and θ. Determine the method of moments estimate of θ.

(a) $\hat{\theta} = 120$
(b) $120 < \hat{\theta} \le 130$
(c) $130 < \hat{\theta} \le 140$
(d) $140 < \hat{\theta} \le 150$
(e) $150 < \hat{\theta}$

$\hat{\theta} = \dfrac{\hat{\sigma}_E^2}{m} = \dfrac{t - m^2}{m}$

$m = \bar{x} = \frac{1}{n} \sum x_i = \frac{340}{5} = 68$

$t = \frac{1}{n} \sum x_i^2 = \frac{65750}{5} = 13150$

$\hat{\theta} = \dfrac{13150 - (68)^2}{68} = \dfrac{8526}{68} = \boxed{125.3824}$

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3) A random variable is suspected of being a distribution with probability density function $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$. You observe the following sample of 11 values from the distribution: 1, 1, 2, 3, 3, 4, 5, 6, 8, 10, and 12. Determine the estimate of λ using percentile matching on the median.

median has $p = 0.5$

(Hint: $X \sim EXP(\theta = 1/\lambda)$.)

(a) 0.10

(b) 0.12

(c) 0.15

(d) 0.17

(e) 0.20

$n = 11, \quad n+1 = 12 \quad \lfloor (n+1)p \rfloor = \lfloor 12(.5) \rfloor = 6$

$j = 6 \quad h = 0$

$\hat{\pi}_{.5} = 4 \overset{set}{=} \pi_{.5} = -\theta \ln(1-p)$

$\hat{\theta} = \frac{-4}{\ln(0.5)}$

so $\hat{\lambda} = \frac{\ln(0.5)}{-4} = \boxed{0.1733}$

$= \frac{1}{\hat{\theta}}$

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4) You believe that a Poisson distribution, with a mean of λ, reflects the number of claims per policy each year. You observe how many claims occur under each of three policies during a year, and compile the following observation data:

Claims per Policy	Number of Observations
0 claims	2
1 claim	1

Calculate the maximum likelihood estimate of λ.

$\hat{\lambda} = \bar{X} = \frac{2(0) + 1(1)}{3} = \boxed{\frac{1}{3} = 0.3333}$

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5) The observations 2, 5, 7, 14, and 20 were obtained as a random sample from a two parameter Pareto distribution with $\alpha = 3$, and unknown parameter θ. a) Estimate θ by percentile matching, using the empirical smoothed estimate of the 75th percentile.

$$n=5, \quad \lfloor (n+1)p \rfloor = \lfloor 6(.75) \rfloor = \lfloor 4.5 \rfloor = 4 = j$$

$$h = 4.5 - 4 = .5$$

$$\hat{\pi}_{.75} = (1-h)\,x_{(j)} + h\,x_{(j+1)} = .5(14+20) = 17 \overset{\text{set}}{=} \pi_{.75}$$

$$17 = \theta\left[(1-.75)^{-\frac{1}{3}} - 1\right], \quad \hat{\theta} = \frac{17}{.5874} = \boxed{28.9410}$$

b) Then use the result from a) to estimate the limited expected value $E[X \wedge d]$ of this distribution at $d = 10$.

$$E[X \wedge 10] = \frac{\theta}{\alpha - 1}\left[1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right] \approx$$

$$\frac{28.941}{2}\left[1 - \left(\frac{28.941}{38.941}\right)^2\right]$$

$$= 14.4705(.4477) = \boxed{6.4777}$$

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e 6) Suppose losses follow an EXP(θ) distribution. Find θ by matching the 88th percentile using the table below. Q3.18

interval (a,b]	proportion
(0,1000]	0.39
(1000,2000]	0.25
(2000,4000]	0.24
(4000,7500]	0.10
(7500,14000]	0.02

$\hat{\pi}_{.88} = 4000 \overset{set}{=} -\theta \ln(1-p)$

$$\hat{\theta} = \frac{4000}{-\ln(0.12)} = \boxed{1886.5579}$$

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7) You are given the following three observations: 0.74 0.81 0.95. You fit a distribution with the following density function to the data: $f(x) = (p+1)x^p$, where $0 < x < 1$ and $p > -1$. Determine the maximum likelihood estimate of p. Q3.16

(A) 4.0
(B) 4.1
(C) 4.2
(D) 4.3
(E) 4.4

$$L(p) = (p+1)^3 [(.74)(.81)(.95)]^p$$

.56943

$$\ln L(p) = 3\ln(p+1) + p \ln[(.74)(.81)(.95)]$$

$$\frac{d \ln L(p)}{dp} = \frac{3}{p+1} + \ln[(.74)(.81)(.95)] \overset{set}{=} 0$$

$$3 = (p+1)[-\ln[(.74)(.81)(.95)]]$$

$$3 = (p+1)(+0.5631)$$

$$\hat{p} = \frac{3}{0.5631} - 1 = \boxed{4.3275}$$

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see back of 3) too

$\ln(.74) + \ln(.81) + \ln(.95) = -0.5631$