for MWF

Math 404 Exam 3 Spring 2016 Name____________________

YOU ARE BEING GRADED FOR WORK

$0.5 \ln(95) = 2.2769$

E3 rev (102)

1) You fit various loss models to 95 claim sizes using maximum likelihood. The fits maximizing the likelihood for a given number of parameters have the following loglikelihoods. Using the Schwarz Bayesian Criterion, what is the selected model? Hint: eliminate one of the 1 parameter models and one of the 2 parameter models by inspection.

closer to 0 is good

| model | number of parameters | maximal loglikelihood | |
|---|---|---|---|
| 1 | 1 | −487 | $\ln(L_r) - \frac{r}{2}\ln(95) =$ −489.2769 |
| 2 | 1 | −487.5 (model 1 is better than 2; bad models) | (−489.7769) |
| 3 | 2 | −487 (model 4 is better than model 3; bad models) | (−491.5538) |
| 4 | 2 | −484.1 | −488.6538 ← |
| 5 | 3 | −482 | −488.8307 |

model 4 with 2 parameters is selected

E3 rev (21)

2) Suppose that the full-credibility standard is defined such that aggregate claim dollars will be within 3% of its true value 95% of the time. The number of claims follows a Poisson distribution, and the claim severity distribution is exponential. Find the indicated full credibility standard in terms of number of claims, using limited fluctuation credibility.

$z_p = 1.96, \quad k = .03, \quad W = S \quad e = n_F$

$CV(X) = 1$ for $Exp(\theta)$.

$$n_0\left(1 + [CV(X)]^2\right) = 2n_0$$

$$= 2\left(\frac{1.96}{.03}\right)^2 = \boxed{8536.8889}$$

$$= 2(4268.4444)$$

easy way: 142) table 5.1.1 127) $v = \alpha\theta$ 132) $a = \alpha\theta^2$ $k = \frac{1}{\theta} = \frac{1}{1.2}$

$E(X) = E[E(X|\lambda)] = E(\lambda) = 1.2$, $\bar{x} = \frac{3+0}{2} = 1.5$

Buhlmann credibility = Bayesian credibility for this model

3) The number of claims made by an individual insured in a year has a Poisson($\lambda$) distribution where $\lambda \sim$ gamma($\alpha = 1, \theta = 1.2$). Three claims are observed in year 1 and no claims are observed in year 2. Using Buhlmann credibility, estimate the number of claims in year 3. Hint: $n = 2, X_1 = 3, X_2 = 0$. $\bar{X} = \frac{3+0}{2} = 1.5$

$\mu(\lambda) = E(X|\lambda) = \lambda = v(\lambda) = V(X|\lambda)$

$EPV = E[V(X|\lambda)] = E(\lambda) = \alpha\theta = 1.2 = v$

$a = VHM = V[E(X|\lambda)] = V(\lambda) = \alpha\theta^2 = 1.44$

$K = \frac{EPV}{VHM} = \frac{v}{a} = \frac{1.2}{(1.2)^2} = \frac{1}{1.2} = 0.8333$, $Z = \frac{n}{n+K} = \frac{2}{2+\frac{1}{1.2}} = 0.7059$

E3 rev see 127

$P = Z\bar{X} + (1-Z)E(X) = E(X) + Z(\bar{X} - E(X))$

$= 1.2 + 0.7059(1.5 - 1.2) = \boxed{1.4118}$

4) The table is used for a sample of 4 claim payments. Use the 4 step Kolmogorov Smirnov test for whether the claims are from an exponential(10) distribution.

E3 rev 96

$F(x) = 1 - e^{-x/10}$

| $x_i$ | $F_n(x_i)$ | $F_n(x_i-)$ | $F^*(x_i)$ | $\max(\lvert F_n(x_i) - F^*(x_i)\rvert, \lvert F_n(x_i-) - F^*(x_i)\rvert)$ | |
|---|---|---|---|---|---|
| 2 | 1/4 | 0/4 | 0.181 | 0.181 | $= .181 - 0$ |
| 5 | 2/4 | 1/4 | 0.393 | 0.143 | $= .393 - .25$ |
| 13 | 3/4 | 2/4 | .727 | 0.227 | $= .727 - .5 \leftarrow D$ |
| 22 | 4/4 | 3/4 | .889 | 0.139 | $= .889 - .75$ |

i) $H_0$ EXP(10) is a good fit $H_1$ not $H_0$

ii) $D = 0.227$

iii) critical value $= \frac{1.36}{\sqrt{n}} = \frac{1.36}{2} = 0.68$

$D < 0.68$, fail to reject $H_0$

iv) the EXP(10) dist is a good fit

(or not enough evidence to conclude EXP(10) is not a good fit)

| total claims $k$ | number of policies $n_k$ |
|---|---|
| 0 | 5000 |
| 1 | 2100 |
| 2 | 750 |
| 3 | 100 |
| 4 | 50 |

$$\bar{X} = \hat{v} = \frac{0 + 2100 + 750(2) + 100(3) + 50(4)}{8000}$$

$$= \frac{4100}{8000} = \frac{41}{80} = 0.5125$$

EBrev 131)

5) During a Year 1, 8000 policies had the above claim experience. Assume the number of claims followed a conditional Poisson($\lambda$) distribution. Assume $\hat{\sigma}_U^2 = 0.5874$. A randomly selected policyholder had 1 claim in Year 1. Determine the semiparametric empirical Bayes estimate of the number of claims in Year 2 for the same policyholder.

$$\hat{a} = \hat{\sigma}_U^2 - \hat{v} = 0.5874 - 0.5125 = 0.0749$$

$$\hat{Z} = \frac{1}{1 + \frac{\hat{v}}{\hat{a}}} = \frac{1}{1 + \underbrace{\frac{.5125}{.0749}}_{K = 6.8425}} = 0.1275$$

$$P_c^1 = \bar{x} + \hat{Z}(x_i - \bar{x}) =$$

$$.5125 + 0.1275(1 - .5125) = \boxed{0.5747}$$

E3 rev 106)

6) Suppose that the claim amount $X$ is uniform$(0, \Theta)$ where the prior distribution of $\Theta$ has pdf $\pi(\theta) = \frac{500}{\theta^2}$ for $\theta > 500$. Two independent claims of 400 and 600 are observed. Find the pdf of the posterior distributions of $\Theta$. Hint: $f(\boldsymbol{x}|\theta) = f(400, 600|\theta) = \frac{1}{\theta}\frac{1}{\theta}$ for $\theta > \max(600, 400) = 600$. Hence the support of the posterior pdf is $\theta > 600$.

$$\pi(\theta|\underline{x}) \propto \pi(\theta) f(\underline{x}|\theta) \propto \theta^{-4}$$

$$\int_{600}^{\infty} c\,\theta^{-4} d\theta = c \left.\frac{\theta^{-3}}{-3}\right|_{600}^{\infty} = c \frac{1}{3(600)^3} = 1$$

so $\boxed{\pi(\theta|\underline{x}) = 3(600)^3\,\theta^{-4} = 648{,}000{,}000\,\theta^{-4} \text{ for } \theta > 600.}$

$$(= 6.48(10^8)\,\theta^{-4})$$

not on 2024 exam 3 $\bar{X} = \frac{9+4+8}{3} = 7$

7) Suppose the number of claims per year for a policyholder follows a Poisson($\lambda$) distribution where $\lambda \sim$ gamma($\alpha = 2.5, \theta = 4$). There were 9,4, and 8 claims in years 1 through 3 for this policyholder. $n\bar{X} = \sum X_i = 21$, $\bar{X} = 7$

(Note: you can get the posterior and predictive distributions from Table 51.1 on the back of p.9 of the exam 3 review.) 132a) was good too 142?

a) Determine the Bayesian credibility estimate (posterior mean) for this policyholder's expected claim frequency in year 4. The answer is $\alpha_* \theta_*$ where $\theta_* = 1/\gamma_*$.

posterior ~ Gamma$(\alpha_*, \theta_*)$ so $\theta_* = \frac{1}{\gamma_*}$, $\gamma_* = 3.25$

posterior mean $= \alpha_* \theta_* =$

$$(\alpha + n\bar{x})\left(\frac{1}{\frac{1}{\theta}+n}\right) = \left(2.5 + 3\,\tfrac{21}{3}\right)\left(\frac{1}{\frac{1}{4}+3}\right)$$

$$= 23.5(0.3077) = 23.5\left(\frac{1}{3.25}\right) = \boxed{7.2308}$$

b) Determine the Bayesian premium for year 4 (the mean of the predictive distribution).

Predictive ~ $NB\left(r = \alpha_*, \beta = \frac{1}{\gamma_*}\right)$

Bayesian premium $= \alpha_* \frac{1}{\gamma_*} = \alpha_* \theta_* = \boxed{7.2308}$

132a) $\alpha' = \alpha + k = \alpha + n\bar{x} = \alpha_*$, $\theta' = \frac{\theta}{1+n\theta} = \theta_*$



Table (p.42) $\alpha_* = \alpha + n\bar{x}$, $\gamma_* = \gamma + n$, $\gamma = \frac{1}{\theta}$, $\gamma_* = \frac{1}{\theta_*}$